

Stat509 Fall 2014 HW1 Solution

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Instruction: Please let me know if you find any error in this solution.

1. Each of the possible five outcomes of a random experiment is equally likely. The sample space is $\{a, b, c, d, e\}$. Let A denote the event $\{a, b\}$, and let B denote the event $\{c, d, e\}$. Determine the following:

(a) $P(A)$

Solution: $P(A) = 2/5$.

(b) $P(B)$

Solution: $P(B) = 3/5$.

(c) $P(\bar{A})$

Solution: $P(\bar{A}) = 1 - P(A) = 1 - 2/5 = 3/5$.

(d) $P(A \cup B)$

Solution: $P(A \cup B) = P(S) = 1$.

(e) $P(A \cap B)$

Solution: $P(A \cap B) = P(\emptyset) = 0$.

2. The probability that a randomly chosen automobile will need an oil change is 0.25; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.14.

- (a) What is the probability that a car will need an oil change or new filter?

Solution: Define $A = \{\text{Need an oil change}\}$ and $B = \{\text{Need new oil filter}\}$. We have $P(A) = 0.25$, $P(B) = 0.4$, and $P(A \cap B) = 0.14$. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.25 + 0.4 - 0.14 = 0.51$.

- (b) If the oil had to be changed, what is the probability that a new oil filter is needed?

Solution: $P(B|A) = P(A \cap B)/P(A) = 0.14/0.25 = 0.56$

- (c) If a new oil filter is needed, what is the probability that the oil has to be changed? **Solu-**

tion: $P(A|B) = P(A \cap B)/P(B) = 0.14/0.4 = 0.35$

- (d) Needing an oil filter and needing an oil change are independent of one another. True or false?

Solution: False, $P(A|B) \neq P(A)$.

3. The use of plant appearance in prospecting for ore deposits is called geobotanical prospecting. One indicator of copper is a small mint with a mauve-colored flower. Suppose that, for a certain region, there is a 30 percent chance that the soil has a high copper content and a 23 percent

chance that the mint will be present there. In addition, we know that if the copper content is high, there is a 70 percent chance that the mint will be present. Let A denote the event that a soil sample has high copper content, and let B denote the event that the mint is present.

- (a) Find the probability that the copper content is not high.

Solution: $P(A) = 0.3$, $P(B) = 0.23$, and $P(B|A) = 0.7$. So, $P(\bar{A}) = 1 - P(A) = 1 - 0.3 = 0.7$.

- (b) Find the probability that the copper content will be high and the mint will be present.

Solution: $P(A \cap B) = P(B|A)P(A) = (0.7)(0.3) = 0.21$.

- (c) Find the probability that the copper content will be high given that the mint is present.

Solution: $P(A|B) = P(A \cap B)/P(B) = 0.21/0.23 = 0.913$.

- (d) Are the events A and B independent? Explain.

Solution: They are not independent, since $P(A|B) \neq P(A)$.

4. A manufacturer of front lights (head lights) for automobiles tests lamps under a high humidity, high temperature environment using intensity and useful life as the responses of interest. The following table shows the performance of 200 lamps.

	Life Good	Satisfactory	Unsatisfactory
Intensity Good	100	25	5
Satisfactory	35	10	5
Unsatisfactory	10	8	2

- (a) What is the probability that a lamp performed unsatisfactorily in useful life?

Solution: There are total 200 lamps. $12/200=0.06$

- (b) What is the probability that a lamp performed unsatisfactorily in intensity?

Solution: $20/200=0.1$

- (c) What is the probability that a lamp performed unsatisfactorily in useful life AND in intensity?

Solution: $2/200=0.01$

- (d) Given that a lamp performed unsatisfactorily in intensity, what is the probability that it performed unsatisfactorily in useful life? **Solution:**

$$P\{\text{poor in useful life} | \text{poor in intensity}\} = \frac{P\{\text{poor in both}\}}{P\{\text{poor in intensity}\}} = 0.01/0.1 = 0.1.$$

5. 75% of a certain part is supplied by vendor A and 25% by vendor B. Vendor A's defect rate is 0.01 and vendor B's defect rate is 0.03.

(a) If a part is from vendor A, what is the probability it is defective?

Solution: Define $D = \{\text{part is defective}\}$, $A = \{\text{part is from vendor A}\}$, and $B = \{\text{part is from vendor B}\}$. $P(D|A) = P(\text{defective rate for vendor A}) = 0.01$.

(b) What is the probability that a randomly chosen part will be defective and from vendor A?

Solution: $P(D \cap A) = P(D|A)P(A) = (0.01)(0.75) = 0.0075$.

(c) What is the probability that a randomly chosen part will be non-defective and from Vendor B?

Solution: $P(\bar{D} \cap B) = P(\bar{D}|B)P(B) = (1 - P(D|B))P(B) = (1 - 0.03)0.25 = 0.2425$.

(d) What is the probability that a randomly chosen part is defective?

Solution: By law of total probability,

$$\begin{aligned} P(D) &= P(D|A)P(A) + P(D|B)P(B) \\ &= (0.01)(0.75) + (0.03)(0.25) \\ &= 0.015 \end{aligned}$$

6. There are 100 light bulbs in a box. 20 are 40 watt, the rest are 60 watt.

(a) If we randomly choose 10 bulbs, what is the probability that there will be exactly 2 40-watt bulbs?

Solution:

$$P(\text{exact 2 40-watt}) = \frac{\binom{20}{2}\binom{80}{8}}{\binom{100}{10}} = 0.318$$

(b) What is the probability that in a random sample of 10 bulbs there will be at least 1 40-watt bulb?

Solution:

$$P(\text{no 40-watt}) = \frac{\binom{20}{0}\binom{80}{10}}{\binom{100}{10}} = 0.095.$$

So,

$$P(\text{at least 1 40 watt}) = 1 - P(\text{no 40-watt}) = 1 - 0.095 = 0.905$$

7. Best Buy gives a choice of 3 CPU models, 2 monitors, 3 printers and 2 scanners. They can operate in any combination. In other words, any CPU can be used with any monitor which works with any printer, etc.

(a) If a configuration contains 1 CPU, 1 monitor, 1 printer and 1 scanner, how many configurations are possible?

Solution: By fundamental theorem of counting, there are $(3)(2)(3)(2)=36$ ways.

- (b) What is the probability of choosing any one random configuration?

Solution: $1/36$

- (c) If the scanner is optional, how many configurations are possible?

Solution: If a scanner is optional, we can choose not to install it. Therefore, there are total

$$(3)(2)(3)(2 + 1) = 54$$

possible ways.

8. A manufacturing operation consists of 10 operations. However, five machining operations must be completed before any of the remaining five assembly operations can begin. Within each set of five, operations can be completed in any order. How many different production sequences are possible?

Solution: The operation can be decomposed into 2 main tasks. For each main tasks, there are five sub-tasks. By fundamental theorem of counting, there are total

$$5! \cdot 5! = 14400$$

possible ways, where $5!$ is the number of ways permuting 5 sub-tasks for each main task.