Stat509 Fall 2014 Homework 5

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1. The time to failure of an electrical component in constant failure rate mode follows an exponential distribution with a mean time to failure of 6000 hours. What is the probability that the average time to failure for 30 randomly chosen components will be less than 5000 hours? (*Hint: use CLT.*)

   Solution:

   Let X denote the time to failure of an electrical component in constant failure rate mode. Then $X \sim exp(1/6000)$ with $\text{E}(X) = 6000$ and $\text{Var}(X) = 6000^2$. The question asks "What is the probability that the average time to failure for 30 randomly chosen components will be less than 5000 hours?", that is, for a random sample with sample size 30, what is the probability that $\overline{X} < 5000$.

   Based on CLT (it is appropriate here since $n > 30$), $\overline{X} \sim \mathcal{AN}(6000, 6000^2/30)$, then we can standardize X to Z and use standard normal table to solve the problem:

$$P(\overline{X} < 5000) = P(Z < \frac{5000 - 6000}{6000/\sqrt{30}}) = P(Z < -0.91) = 0.1814$$

2. A manufacturing company produces water filters for home refrigerators. They want to estimate the defective rate p. A random sample of 300 filters yielded 7 defects.

   (a) Find a point estimate of p.

   Solution: A point estimator of p is $\hat{p}$, the sample proportion. Therefore, a point estimate of p is $\hat{p} = 7/300 = 0.023$

   (b) Estimate the defective rate with a 99% confidence interval. Interpret the confidence interval using the context of the question.

   Solution: Recall that the $100(1 - \alpha)\%$ CI for p has the following form:

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

   For a 99% confidence level, $\alpha = 0.01$ and $z_{0.005} = 2.58$. Therefore the 99% CI is:

$$0.023 \pm 2.58 \times \sqrt{\frac{0.023(1 - 0.023)}{300}} = (0.0007, 0.0453).$$

   Interpretation: with 99% confidence, the population defective rate for water filters is between 0.0007 and 0.0453.

3. In a random sample of 85 automobile engine crankshaft bearings, 7 have a surface finish roughness that exceeds the specifications. Does this data present sufficient evidence that the proportion of crankshaft bearings, say p, exhibiting excess surface roughness is greater than 0.05? We will address this using a hypothesis test.

   (a) State the null and alternative hypotheses.

Solution:

$$H_0 : p = 0.05$$
$$H_a : p > 0.05$$

(b) Calculate the appropriate test statistic.
Solution: Note that $\hat{p} = 7/85 = 0.082$

$$z_0 = \frac{0.082 - 0.05}{\sqrt{\frac{0.05(1-0.05)}{85}}} = 1.35$$

(c) What is the p-value of the test?
Solution: p-value $= P(Z > z_0) = P(Z > 1.35) = 0.0885$

(d) What is your conclusion based on the p-value if we use $\alpha = 0.1$?
Solution: Our p-value=0.0885 $< \alpha$, therefore we reject H_0, and conclude that the proportion of crankshaft bearings exhibiting excess surface roughness is greater than 0.05 (note that this is H_a in the context of the question).

(e) What is the probability of a Type I error? Assume $\alpha = 0.1$.
Solution: P(Type I error)= $\alpha = 0.1$.

(f) Calculate a 90% confidence interval for the population proportion.
Solution:

$$0.082 \pm 1.64\sqrt{\frac{0.082(1-0.082)}{85}} = (0.033, 0.131)$$

4. In question 3, our interest is to see whether p is greater than 0.05. Under the same setting ($\hat{p} = 7/85 = 0.082$), suppose the researcher's interest now changes to whether the population proportion is 0.05 or not (Hint: this denotes a two-tail test). Redo part (a)-(e) in question 3.

(a) State the null and alternative hypotheses.
Solution:

$$H_0 : p = 0.05$$
$$H_a : p \neq 0.05$$

(b) Calculate the appropriate test statistic.
Solution: Note that $\hat{p} = 7/85 = 0.082$

$$z_0 = \frac{0.082 - 0.05}{\sqrt{\frac{0.05(1-0.05)}{85}}} = 1.35$$

(c) What is the p-value of the test?
Solution: p-value $= 2P(Z < -|z_0|) = 2P(Z < -1.35) = 0.177$

(d) What is your conclusion based on the p-value if we use $\alpha = 0.1$?
Solution: Our p-value=0.177 $> \alpha$, therefore we do not reject H_0, and we do not have enough evidence to conclude that the proportion of crankshaft bearings exhibiting excess surface roughness is not equal to 0.05.

(e) What is the probability of a Type I error? Assume $\alpha = 0.1$.
Solution: The probability of Type I error does not change with the type of hypothesis. It is controlled by the experimenter (through the value of α). In this question, P(Type I error)$= \alpha = 0.1$.