95% CI for Type 1 Error Rate (Homework 4 Q3)

Let X_i represent whether or not we reject the null hypothesis for the ith simulation iteration, $i = 1, 2, ...m$ under the significance level α.
$X_i = 1$ if we reject the null hypothesis; $X_i = 0$ otherwise. Then,

$$X_i \sim Bernoulli(P_r), P_r = \alpha$$

Hence $E(X_i) = P_r$, $Var(X_i) = P_r(1 - P_r)$.
The empirical type I error rate is calculated as: $\overline{X}$
According to the CLT, when m is large enough,

$$\overline{X} \sim N(P_r, \frac{P_r(1 - P_r)}{m})$$

The 95% CI for the true type I error rate can be calculated as:

$$\overline{X} \pm Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\widehat{P_r}(1 - \widehat{P_r})}{m}}$$

$\widehat{P_r} = \overline{X}$