

# Fixed vs. Random Effects

Department of Statistics, University of South Carolina

Stat 705: Data Analysis II

# Outline

- Review One-Way ANOVA
- Fixed vs. Random Effects
- Random Effects

## ANOVA Review (KNN Chap 16)

Analysis of variance (ANOVA) models are regression models with qualitative predictors, called factors or treatments.

Factors have different levels.

For example, the factor “education” may have the levels *high school*, *undergraduate*, *graduate*. The factor “gender” has two levels *female*, *male*.

We may have several factors as predictors, e.g. race and gender may be used to predict annual salary in \$.

## 16.3 Cell means model

Have  $r$  different treatments or factor levels. At each level  $i$ , have  $n_i$  observations from group  $i$ .

Total number of observations is  $n_T = n_1 + n_2 + \cdots + n_r$ .

Response is  $Y_{ij}$  where  $\left\{ \begin{array}{l} i = 1, \dots, r \text{ factor level} \\ j = 1, \dots, n_i \text{ obs. within factor level} \end{array} \right\}$ .

Example: Two factors: MS, PhD.  $Y_{ij}$  is age in years. Spring of 2014 we observe

$$Y_{11} = 28, Y_{12} = 24, Y_{13} = 24, Y_{14} = 22, Y_{15} = 26, Y_{16} = 23,$$

$$Y_{21} = 29, Y_{22} = 23, Y_{23} = 26, Y_{24} = 25, Y_{25} = 22, Y_{26} = 23, Y_{27} = 38, Y_{28} = 33, Y_{29} = 30, Y_{2,10} = 27.$$

# One-way ANOVA model

$$Y_{ij} = \mu_i + \epsilon_{ij}, \quad \epsilon_{ij} \stackrel{iid}{\sim} N(0, \sigma^2).$$

Can rewrite as

$$Y_{ij} \stackrel{ind.}{\sim} N(\mu_i, \sigma^2).$$

- Data are normal, data are independent, variance constant across groups.
- $\mu_i$  is allowed to be different for each group.  $\mu_1, \dots, \mu_r$  are the  $r$  population means of the response. A picture helps.
- Questions: what is  $E\{Y_{ij}\}$ ? What is  $\sigma^2\{Y_{ij}\}$ ?

# Matrix formulation

(pp. 683–684, 710–712 in KNN) For  $r = 3$  we have

$$\begin{bmatrix} Y_{11} \\ Y_{12} \\ \vdots \\ Y_{1n_1} \\ Y_{21} \\ Y_{22} \\ \vdots \\ Y_{2n_2} \\ Y_{31} \\ Y_{32} \\ \vdots \\ Y_{3n_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} + \begin{bmatrix} \epsilon_{11} \\ \epsilon_{12} \\ \vdots \\ \epsilon_{1n_1} \\ \epsilon_{21} \\ \epsilon_{22} \\ \vdots \\ \epsilon_{2n_2} \\ \epsilon_{31} \\ \epsilon_{32} \\ \vdots \\ \epsilon_{3n_3} \end{bmatrix}$$

or

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}.$$

## 16.4 Fitting the model

For  $r = 3$ , let  $Q(\mu_1, \mu_2, \mu_3) = \sum_{i=1}^3 \sum_{j=1}^{n_i} (Y_{ij} - \mu_i)^2$ .

Need to minimize this over all possible  $(\mu_1, \mu_2, \mu_3)$  to find least-squares (LS) solution. Can easily show that  $Q(\mu_1, \mu_2, \mu_3)$  has minimum at

$$\hat{\beta} = \begin{bmatrix} \hat{\mu}_1 \\ \hat{\mu}_2 \\ \hat{\mu}_3 \end{bmatrix} = \begin{bmatrix} \bar{Y}_{1.} \\ \bar{Y}_{2.} \\ \bar{Y}_{3.} \end{bmatrix}$$

where  $\bar{Y}_{i.} = \frac{1}{n_i} \sum_{j=1}^{n_i} Y_{ij}$  is the sample mean from the  $i$ th group (pp. 687–688).

These  $\hat{\beta}$  are also maximum likelihood estimates.

# Matrix formula of least-squares estimators ( $r = 3$ )

$$\mathbf{x}'\mathbf{x} = \begin{bmatrix} 1 & \cdots & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_1 & 0 & 0 \\ 0 & n_2 & 0 \\ 0 & 0 & n_3 \end{bmatrix},$$

$$(\mathbf{x}'\mathbf{x})^{-1} = \begin{bmatrix} n_1^{-1} & 0 & 0 \\ 0 & n_2^{-1} & 0 \\ 0 & 0 & n_3^{-1} \end{bmatrix}, \quad \mathbf{x}'\mathbf{y} = \begin{bmatrix} Y_{1.} \\ Y_{2.} \\ Y_{3.} \end{bmatrix},$$

$$\Rightarrow \hat{\boldsymbol{\beta}} = (\mathbf{x}'\mathbf{x})^{-1}\mathbf{x}'\mathbf{y} = \begin{bmatrix} \bar{Y}_{1.} \\ \bar{Y}_{2.} \\ \bar{Y}_{3.} \end{bmatrix}.$$



As in regression (STAT 704),

$$e_{ij} = Y_{ij} - \hat{Y}_{ij} = Y_{ij} - \hat{\mu}_i = Y_{ij} - \bar{Y}_i.$$

As usual,  $\hat{Y}_{ij}$  is the estimated mean response under the model.

Note that  $\sum_{j=1}^{n_i} e_{ij} = 0$ ,  $i = 1, \dots, r$ . [check this!]

In matrix terms

$$\mathbf{e} = \mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{Y} - \hat{\mathbf{Y}}.$$

## 16.5 ANOVA table (pp. 690–698)

Define the following

$$Y_{i.} = \sum_{j=1}^{n_i} Y_{ij} = i \text{ group sum,}$$

$$\bar{Y}_{i.} = \frac{1}{n_i} \sum_{j=1}^{n_i} Y_{ij} = i\text{th group mean}$$

$$Y_{..} = \sum_{i=1}^r \sum_{j=1}^{n_i} Y_{ij} = \sum_{i=1}^r Y_{i.} = \text{sum all obs.}$$

$$\bar{Y}_{..} = \frac{1}{n_T} \sum_{i=1}^r \sum_{j=1}^{n_i} Y_{ij} = \frac{1}{n_T} \sum_{i=1}^r Y_{i.} = \text{mean all obs.}$$

## Sums of squares for treatments, error, and total

$$\text{SSTO} = \sum_{i=1}^r \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{..})^2 = \text{variability in } Y_{ij}'\text{s}$$

$$\text{SSTR} = \sum_{i=1}^r \sum_{j=1}^{n_i} (\hat{Y}_{ij} - \bar{Y}_{..})^2 = \sum_{i=1}^r \sum_{j=1}^{n_i} (\hat{\mu}_{ij} - \bar{Y}_{..})^2$$

$$= \sum_{i=1}^r \sum_{j=1}^{n_i} (\bar{Y}_{i.} - \bar{Y}_{..})^2 = \sum_{i=1}^r n_i (\bar{Y}_{i.} - \bar{Y}_{..})^2$$

= variability explained by ANOVA model

$$\text{SSE} = \sum_{i=1}^r \sum_{j=1}^{n_i} (Y_{ij} - \hat{Y}_{ij})^2 = \sum_{i=1}^r \sum_{j=1}^{n_i} e_i^2$$

= variability NOT explained by ANOVA model

- As before in regression,

$$\underbrace{\text{SSTO}}_{\text{total}} = \underbrace{\text{SSTR}}_{\text{treatment effects}} + \underbrace{\text{SSE}}_{\text{leftover randomness}}$$

- $\text{SSE}=0 \Rightarrow Y_{ij} = Y_{ik}$  for all  $j \neq k$  and for  $i = 1, \dots, r$ .
- $\text{SSTR}=0 \Rightarrow \bar{Y}_{i\cdot} = \bar{Y}_{\cdot\cdot}$  for  $i = 1, \dots, r$ .

## ANOVA table (p. 694)

| Source | SS  | df        | MS                | E(MS)  |
|--------|---|-----------|-------------------|--|
| SSTR   | $\sum_{i=1}^r \sum_{j=1}^{n_i} (\bar{Y}_{i.} - \bar{Y}_{..})^2$ | $r - 1$   | $SSTR / (r - 1)$  | $\sigma^2 + \frac{\sum_{i=1}^r n_i (\mu_i - \mu_{..})^2}{r - 1}$ |
| SSE    | $\sum_{i=1}^r \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{i.})^2$       | $n_T - r$ | $SSE / (n_T - r)$ | $\sigma^2$   |
| SSTO   | $\sum_{i=1}^r \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{..})^2$       | $n_T - 1$ |                   |  |

## Degrees of freedom

- SSTO has  $n_T - 1$  df because there are  $n_T$   $Y_{ij} - \bar{Y}_{..}$  terms in the sum, but they add up to zero (1 constraint).
- SSE has  $n_T - r$  df because there are  $n_T$   $Y_{ij} - \bar{Y}_{i.}$  terms in the sum, but there are  $r$  constraints of the form  $\sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{i.}) = 0$ ,  $i = 1, \dots, r$ .
- SSTR has  $r - 1$  df because there are  $r$  terms  $n_i(\bar{Y}_{i.} - \bar{Y}_{..})$  in the sum, but they sum to zero (1 constraint).

Assuming  $\mu_1 = \dots = \mu_r$ , Cochran's Theorem (Section 2.7) shows that  $SSTR/\sigma^2 \sim \chi_{r-1}^2$  and  $SSE/\sigma^2 \sim \chi_{n_T-r}^2$  and they are independent.

## Expected mean squares

$E\{\text{MSE}\} = \sigma^2$ , MSE is unbiased estimate of  $\sigma^2$

$$E\{\text{MSTR}\} = \sigma^2 + \frac{\sum_{i=1}^r n_i (\mu_i - \mu_{\cdot})^2}{r - 1},$$

where  $\mu_{\cdot} = \sum_{i=1}^r \frac{n_i \mu_i}{n_T}$  is the weighted average of  $\mu_1, \dots, \mu_r$  (pp. 696–698).

If  $\mu_i = \mu_j$  for all  $i, j \in \{1, \dots, r\}$  then  $E\{\text{MSTR}\} = \sigma^2$ , otherwise  $E\{\text{MSTR}\} > \sigma^2$ .

Hence, if any group means are different then  $\frac{E\{\text{MSTR}\}}{E\{\text{MSE}\}} > 1$ .

## 16.6 F test of $H_0 : \mu_1 = \cdots = \mu_r$

Fact: If  $\mu_1 = \cdots = \mu_r$  then

$$F^* = \frac{\text{MSTR}}{\text{MSE}} \sim F(r-1, n_T - r).$$

To perform  $\alpha$ -level test of  $H_0 : \mu_1 = \cdots = \mu_r$  vs.  $H_a$  : some  $\mu_i \neq \mu_j$  for  $i \neq j$ ,

- Accept if  $F^* \leq F(1 - \alpha, r - 1, n_T - r)$  or p-value  $\geq \alpha$ .
- Reject if  $F^* > F(1 - \alpha, r - 1, n_T - r)$  or p-value  $< \alpha$ .

p-value =  $P\{F(r - 1, n_T - 1) \geq F^*\}$ .



- If  $r = 2$  then  $F^* = (t^*)^2$  where  $t^*$  is t-statistic from 2-sample pooled-variance t-test.
- The F-test may be obtained from the general nested linear hypotheses approach (big model / little model). Here the full model is  $Y_{ij} = \mu_i + \epsilon_{ij}$  and the reduced is  $Y_{ij} = \mu + \epsilon_{ij}$ .

$$F^* = \frac{\left[ \frac{SSE(R) - SSE(F)}{dfE_R - dfE_F} \right]}{\frac{SSE(F)}{dfE_F}} = \frac{MSTR}{MSE}.$$

## 16.7 Alternative formulations

An alternative formula for ANOVA model can be written as:

$$Y_{ij} = \mu + \alpha_i + \epsilon_{ij},$$

where  $\alpha_r = 0$ .

- $E\{Y_{rj}\} = \mu$ ;  $\mu$  is the cell-mean for the  $r$ th level.
- For  $i < r$ ,  $E\{Y_{ij}\} = \mu + \alpha_i$ ;  $\alpha_i$  is  $i$ 's offset to group  $r$ 's mean  $\mu$ .

Can we express this formulation using a linear regression model?

## Fixed vs. Random Effects

- In ANOVA, the categorical variables are well-defined categories: such as genotype groups, age groups
- In some designs, the categorical variable is “subject.”
- Simplest example: repeated measures, where more than one measurement is taken on the same individual.
- In this case, the “group” effect  $\alpha_j$  is best thought of as random because we only sample a subset of the entire population of subjects.

## When to use random effects

- A “group” effect is random if we can think of the levels we observe in the group to be samples from a larger population.
- Example: if collecting data from different medical centers, “center” might be thought of as random.
- Example: if surveying students on different campuses, “campus” may be a random effect.

## Examples: Sodium content in beer

- How much sodium is there in North American beer? How much does this vary by brand?
- Observations: for 6 brands of beer, researchers recorded the sodium content of 8 12-ounce bottles
- Questions of interest: what is the “grand mean” sodium content? How much variability is there from brand to brand?
- Since brand was sampled from a larger set, we can think of various brands as random variables.
- “Individuals” in this case are brands, repeated measured 8 times.

## 25.1 One-way random cell means model

If treatment levels come from a larger population, their effects are best modeled as random. A one-way random cell means model is

$$Y_{ij} = \mu_i + \epsilon_{ij},$$

where

$$\mu_1, \dots, \mu_r \stackrel{iid}{\sim} N(\mu., \sigma_\mu^2) \text{ independent of } \epsilon_{ij} \stackrel{iid}{\sim} N(0, \sigma^2).$$

As usual,  $i = 1, \dots, r$  and  $j = 1, \dots, n_i$ .

The test of interest is  $H_0 : \sigma_\mu^2 = 0$ .

We can re-express the model as a random effects model, by writing  $\mu_i = \mu. + \tau_i$ , where  $\tau_1, \dots, \tau_r \stackrel{iid}{\sim} N(0, \sigma_\mu^2)$ .

$\tau_1, \dots, \tau_r$  are called *random effects* and  $\sigma_\mu^2$  and  $\sigma^2$  are termed *variance components*. This model is an example of a *random effects* model, because it has only random effects beyond the intercept  $\mu.$  (which is fixed).

The random cell means model has some quite different properties from the fixed cell means model.

- 1  $E(Y_{ij}) = \mu.$
- 2  $\sigma^2 \{Y_{ij}\} = \sigma^2 + \sigma_\mu^2$  (Hence the term *variance components*)
- 3  $\sigma \{Y_{ij}, Y_{ij'}\} = \sigma_\mu^2$
- 4  $\rho \{Y_{ij}, Y_{ij'}\} = \frac{\sigma_\mu^2}{\sigma^2 + \sigma_\mu^2}$
- 5  $E(\bar{Y}_{..}) = \mu.$
- 6  $\sigma^2(\bar{Y}_{..}) = \frac{\sigma^2 + n\sigma_\mu^2}{rn}$

# Variance-Covariance Matrix

- Suppose  $r=2$  levels, and  $n=2$  cases in each level. The observation vector is:

$$\mathbf{Y} = \begin{bmatrix} Y_{11} \\ Y_{12} \\ Y_{21} \\ Y_{22} \end{bmatrix} .$$

- The variance-covariance matrix of  $\mathbf{Y}$  is:



# Variance-Covariance Matrix

- Suppose  $r=2$  levels, and  $n=2$  cases in each level. The observation vector is:

$$\mathbf{Y} = \begin{bmatrix} Y_{11} \\ Y_{12} \\ Y_{21} \\ Y_{22} \end{bmatrix}.$$

- The variance-covariance matrix of  $\mathbf{Y}$  is:

$$\text{Cov}(\mathbf{Y}) = \begin{bmatrix} \sigma^2 & \sigma_{\mu}^2 & 0 & 0 \\ \sigma^2 & \sigma_{\mu}^2 & 0 & 0 \\ 0 & 0 & \sigma^2 & \sigma_{\mu}^2 \\ 0 & 0 & \sigma^2 & \sigma_{\mu}^2 \end{bmatrix}.$$

## Simple random effect model

$$Y_{ij} = \mu_i + \epsilon_{ij},$$

where

$$\mu_1, \dots, \mu_r \stackrel{iid}{\sim} N(\mu., \sigma_\mu^2)$$

independent of

$$\epsilon_{ij} \stackrel{iid}{\sim} N(0, \sigma^2).$$

As usual,  $i = 1, \dots, r$  and  $j = 1, \dots, n_i$ .

- We might be interested in the population mean  $\mu$ : CI, is it zero?
- What is really usually the focus:  $\sigma_\mu$ : CI, is it zero?

# ANOVA table for one-way random effect

| Source | SS  | df        | MS                | E(MS)                      |
|--------|---|-----------|-------------------|----------------------------|
| SSTR   | $\sum_{i=1}^r \sum_{j=1}^{n_i} (\bar{Y}_{i.} - \bar{Y}_{..})^2$ | $r - 1$   | $SSTR / (r - 1)$  | $\sigma^2 + n\sigma_\mu^2$ |
| SSE    | $\sum_{i=1}^r \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{i.})^2$       | $n_T - r$ | $SSE / (n_T - r)$ | $\sigma^2$                 |
| SSTO   | $\sum_{i=1}^r \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{..})^2$       | $n_T - 1$ |                   |                            |

- Only change here is the expectation of MSTR reflects the randomness of  $\mu_j$ s
- Under  $H_0 : \sigma_\mu^2 = 0$ , it is easy to see that

$$\frac{MSTR}{MSE} \sim F_{(r-1), (n-1)r}$$

## Inference for $\mu$

- We know that  $E(\overline{Y_{..}}) = \mu$  and  $\sigma^2(\overline{Y_{..}}) = \frac{n\sigma_{\mu}^2 + \sigma^2}{rn}$ .
- Therefore,

$$\frac{\overline{Y_{..}} - \mu}{\sqrt{\frac{SSTR}{(r-1)rn}}} \sim t_{r-1}$$

- Why  $r - 1$  degree of freedom? If we can sample infinite number of observations from each level, so that  $\overline{Y_{i.}} \rightarrow \mu_i$
- To learn anything about  $\mu$ , we still only have  $r$  observations ( $\mu_1, \mu_2, \dots, \mu_r$ )
- Hence, sampling more within the “group” can not narrow the CI for  $\mu$ .

## Testing $H_0 : \sigma_\mu = 0$

The MSE and MSTR are defined as they were before. One can show  $E(MSE) = \sigma^2$  and  $E(MSTR) = \sigma^2 + n\sigma_\mu^2$  when  $n = n_i$  for all  $i$ . Most packages provides symbolic forms of expected mean squares for random/mixed models if requested.

If  $\sigma_\mu = 0$  we expect  $F^* = MSTR/MSE$  to be somewhat larger than 1. In fact, just like the fixed-effects case,  $F^* \sim F(r - 1, n_T - r)$ . This is the test given by `proc glm` when you add a `random A;` statement.

One can also fit the model in `proc mixed`, but this procedure provides a slightly cruder test of  $H_0 : \sigma_\mu = 0$ .

## Other tests and estimates

We can derive estimates for  $\mu$ ,  $\sigma^2$  and  $\frac{\sigma_\mu^2}{\sigma^2 + \sigma_\mu^2}$  because pivotal quantities are readily available. It is an open question whether we are interested in inference on  $\mu$  in most practical applications.

Other quantities of interest tended to require moment-based estimates (old school)—e.g., the variance component  $\sigma_\mu^2$ . Methods to provide point estimates and/or standard errors include

- Maximum Likelihood (biased)
- Restricted Maximum Likelihood

# Musical competition

$r = 4$  judges were selected at random to review students' musical performances on trumpet. Four students were randomly selected to have their performances reviewed by a given judge.  $Y_{ij}$  is the rating of the  $i$ th judge on the  $j$ th student.

Since the judges are chosen randomly from a large population of available judges, the random-effects one-way model applies.

```
data music;
input rating judge @@;
datalines;
  76 1 65 1 85 1 74 1 59 2 75 2 81 2 67 2
  49 3 63 3 61 3 46 3 74 4 71 4 85 4 89 4
;
run;

proc glm; class judge; * Chapter 16, fixed-effects approach;
  model rating=judge; run;

proc glm; class judge; * Chapter 25, mixed-effects approach;
  model rating=judge;
  random judge; run;

proc glimmix; class judge; * preferred procedure;
  model rating= / s cl;
  random judge / g cl;
  covtest zerog; * tests H0: sigma_mu=0 vs. H0: sigma_mu>0;
```

## 25.2 Random factor effects and mixed factor effects

In the two-factor random effects model, we specify

$$Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$$

where  $\mu_{..}$  is a constant,  $\alpha_i \stackrel{iid}{\sim} N(0, \sigma_\alpha^2)$ ,  $\beta_j \stackrel{iid}{\sim} N(0, \sigma_\beta^2)$ ,  $(\alpha\beta)_{ij} \stackrel{iid}{\sim} N(0, \sigma_{\alpha\beta}^2)$  and  $\epsilon_{ijk} \stackrel{iid}{\sim} N(0, \sigma^2)$  are pairwise independent.

As mentioned in class, these models can be useful when studying sources of measurement error, e.g., in an industrial R & R study (Repeatability and Reproducibility).



## 25.2 Random factor effects and mixed factor effects

The Expected Mean Squares suggests a different set of F tests to test variance components.

| <u>Source</u> | <u>df</u>  | <u>EMS</u>   |
|---------------|------------|--|
| A             | a-1        | $\sigma^2 + n\sigma_{\alpha\beta}^2 + nb\sigma_{\alpha}^2$ |
| B             | b-1        | $\sigma^2 + n\sigma_{\alpha\beta}^2 + na\sigma_{\beta}^2$  |
| AB            | (a-1)(b-1) | $\sigma^2 + n\sigma_{\alpha\beta}^2$                       |
| Error         | ab(n-1)    | $\sigma^2$   |
| Total         | abn-1      |  |

## 25.2 Drug case study

On-line prices for several drugs were checked at multiple local pharmacies. Is this a random effects two-factor study? Are model assumptions satisfied?

|           | Medicine |        |        |        |        |        |
|-----------|----------|--------|--------|--------|--------|--------|
|           | Walmart  | Mart   | Target | Longs  | Agape  | CVS    |
| Lasix     | 29.76    | 31.52  | 31.52  | 31.71  | 32.22  | 31.92  |
| Nardil    | 18.50    | 23.13  | 24.54  | 23.23  | 24.89  | 17.62  |
| Procardia | 23.25    | 25.53  | 32.88  | 25.53  | 29.62  | 23.72  |
| Naprosyn  | 187.58   | 184.75 | 185.81 | 185.45 | 185.45 | 188.90 |
| Danazol   | 139.39   | 135.30 | 148.19 | 135.40 | 251.02 | 153.30 |
| Lidocaine | 239.75   | 157.24 | 216.36 | 157.14 | 157.14 | 145.07 |

## 25.2 Drug case study

```
data pharmacy;
length Drug $10;
input Drug $ Pharmacy $ Price;
datalines;
Lasix Walmart 29.76 Lasix MedMart 31.52 & Lasix Target 31.52 Lasix Longs 31.71
Lasix Agape 32.22 Lasix CVS 31.92
...
Lidocaine Walmart 239.75 Lidocaine MedMart 157.24 Lidocaine Target 216.36
Lidocaine Longs 157.14 Lidocaine Agape 157.14 Lidocaine CVS 145.07
;
run;

proc glm data=pharmacy; class drug pharmacy;
model price=drug pharmacy;
random drug pharmacy/test; run;

proc glimmix data=pharmacy; class drug pharmacy;
model price= / s cl;
random drug pharmacy / g cl;
covtest zerog;
run;
```