

Homework 10

STAT 509 Statistics for Engineers

Summer 2017 Section 001

Instructor: Tahmidul Islam

1. Suppose $X \sim \text{Poisson}(\lambda)$ and we want to find the value of λ . We collect a sample of 20 observations:

$\{4, 2, 1, 3, 5, 3, 0, 2, 3, 1, 4, 2, 2, 2, 2, 2, 0, 4, 4, 1\}$

- (a) Because $E(X) = \lambda$, it is reasonable to use **sample mean** as a point estimator to estimate the value of λ (population mean). Use R to get the value of the estimate.
- (b) Because X is a Poisson random variable, we have $\text{Var}(X) = \lambda$. Therefore, it is also reasonable to use sample variance as a point estimator to estimate the value of λ (population variance). Use R to get the value of the estimate.
- (c) Some day the oracle tells you the true value of λ is 2.5. Which estimate is better in this case, sample mean or sample variance?
2. An electrical component's lifetime follows an exponential distribution with a mean time to failure of 6000 hours.
- (a) n components are randomly chosen, what is the asymptotic distribution for the average time to failure for these n components? (Hint: Central Limit Theorem)
- (b) What is the probability that the average time to failure for 500 randomly chosen components will be less than 5800 hours? (Hint: Sample size 500 is considered large enough to use CLT)
3. Shiwen decides to eat some chocolates, which has p chance to be made by Carolina Reaper. p is unknown. Shiwen randomly eats 8 chocolates, in which 1 of them makes him suffer.
- (a) Find a point estimate of p .
- (b) What is the standard error of the point estimator \hat{p} ?
- (c) Shiwen is not satisfied with the precision level of this result. By the end of the day, he eats 100 chocolates, in which 6 make him suffer. Find a point estimate of p using 100 observations, and find the standard error. Is the standard error smaller?