Homework 10 Solution

STAT 509 Statistics for Engineers
Summer 2017 Section 001
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1. Suppose $X \sim Poisson(\lambda)$ and we want to find the value of λ. We collect a sample of 20 observations:

$$\{4, 2, 1, 3, 5, 3, 0, 2, 3, 1, 4, 2, 2, 2, 2, 2, 0, 4, 4, 1\}$$

   (a) Because $E(X) = \lambda$, it is reasonable to use **sample mean** as a point estimator to estimate the value of λ (population mean). Use R to get the value of the estimate.

   (b) Because X is a Poisson random variable, we have $Var(X) = \lambda$. Therefore, it is also reasonable to use sample variance as a point estimator to estimate the value of λ (population variance). Use R to get the value of the estimate.

   (c) Some day the oracle tells you the true value of λ is 2.5. Which estimate is better in this case, sample mean or sample variance?

   Solution:

   (a) Sample mean $\bar{x} = \frac{\sum_i x_i}{n}$.

```
x <- c(4,2,1,3,5,3,0,2,3,1,4,2,2,2,2,2,0,4,4,1)
mean(x)
[1] 2.35
```

   (b) Sample variance $s^2 = \frac{\sum_i (x_i - \bar{x})^2}{n-1}$.

```
sd(x)
[1] 1.386969
```

   (c) $\bar{x} - \lambda = 2.35 - 2.5 = -0.15$ and $s^2 - \lambda = 1.386969 - 2.5 = -1.113031$. Sample mean is closer to the true value of λ. So sample mean is better estimator in this case.

2. An electrical component's lifetime follows an exponential distribution with a mean time to failure of 6000 hours.

   (a) n components are randomly chosen, what is the asymptotic distribution for the average time to failure for these n components? (Hint: Central Limit Theorem)

   (b) What is the probability that the average time to failure for 500 randomly chosen components will be less than 5800 hours? (Hint: Sample size 500 is considered large enough to use CLT)

   Solution:

   Let X = Failure time. $X \sim exp(\lambda = 1/6000)$

   (a) Let $\bar{X}$ = Sample mean (average) to failure. By central limit theorem we know, $\bar{X} \sim AN(\mu = 6000, 6000^2/n)$ for large n.

   (b) Since n = 500 is large enough to apply CLT, we write $\bar{X} \sim AN(6000, 6000^2/500)$. Now, $P(\bar{X} < 5800)$ can be calculated directly.

```
#P(Xb < 5800)
pnorm(5800, 6000, 6000/sqrt(500))
[1]  0.2280283
```

3. Shiwen decides to eat some chocolates, which has p chance to be made by Carolina Reaper. p is unknown. Shiwen randomly eats 8 chocolates, in which 1 of them makes him suffer.

   (a) Find a point estimate of p.

   (b) What is the standard error of the point estimator $\hat{p}$?

   (c) Shiwen is not satisfied with the precision level of this result. By the end of the day, he eats 100 chocolates, in which 6 make him suffer. Find a point estimate of p using 100 observations, and find the standard error. Is the standard error smaller?

   Solution:

   X = Number of Reaper eaten. n = Total number of chocolate eaten.

   (a) $\hat{p} = \frac{X}{n} = \frac{1}{8} = 0.125$

   (b) Standard error of $\hat{p} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.125\times0.875}{8}} = 0.0137$.

   (c) $\hat{p} = \frac{X}{n} = \frac{6}{100} = 0.06$.

   Standard error of $\hat{p} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.06\times0.94}{100}} = 0.000564$. Yes, the standard error became smaller.