Homework 11 Solution

STAT 509 Statistics for Engineers
Summer 2017 Section 001
Instructor: Tahmidul Islam

1. In a random sample of 85 automobile engine crankshaft bearings, 7 have a surface finish roughness that exceeds the specifications. Does this data present sufficient evidence that the proportion of crankshaft bearings, say p, exhibiting excess surface roughness is greater than 0.06? We will address this using a hypothesis test.

   (a) State the null and alternative hypotheses.

   (b) Calculate the appropriate test statistic.

   (c) What is the p-value of the test?

   (d) What is your conclusion based on the p-value if we use $\alpha = 0.1$?

   (e) Calculate a 90% confidence interval for the population proportion. Interpret the confidence interval using the context of the question.

   (f) Compare the results in (d) and (e), are they similar?

   (g) What is the probability to make type I error?

   Solution:

   Let p = proportion of bearings exceeding surface roughness specification.

   (a) $H_0 : p = 0.06$ vs. $H_a : p > 0.06$.

   (b) Sample proportion $\hat{p} = \frac{7}{85} = 0.082$. Test statistics

   $$z_0 = \frac{0.082 - 0.06}{\sqrt{\frac{0.06(1-0.06)}{85}}} = 0.8677699.$$

   (c) P-value $= P(Z > 0.8677699) = 1 - P(Z < 0.8677699) = 1 - 0.8078 = 0.1922$.

   (d) Here p-value $= 0.19 > 0.10$. Therefore, we failed to reject H_0 and conclude that we do not have sufficient evidence that the proportion of crankshaft bearings exhibiting excess surface roughness is greater than 0.06.

   (e) Margin of error:

   $$\begin{aligned} & z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \\ &= z_{0.05}\sqrt{\frac{0.082(1-0.082)}{85}} \\ &= 1.64 \times 0.02981727 \\ &= 0.04890032. \end{aligned}$$

   So, 90% CI for the population proportion is $0.082 \pm 0.049 = (0.033, 0.131)$. We are 90% confident that the population proportion of crankshaft bearings exhibiting excess surface roughness is atleast 3.3% and atmost 13.1%.
   Or,
   We are 90% confident that atleast 3.3% and atmost 13.1% of all crankshaft bearings

exhibiting excess surface roughness.
More rigorously,
95% of the time, the population proportion of crankshaft bearings exhibiting excess surface roughness is between 3.3% and 13.1%.

(f) Confidence interval provides range of possible values of the parameter. Since 0.06 is in the range (0.033, 0.131), it a plausible value of the proportion. The conclusion from hypothesis testing is same, we can not say the proportion is larger than 0.06, so 0.06 is possible.

(g) $P(\text{type I error}) = \alpha = 0.1$.

2. You plan to hold a party for your friends, and you are interested to know at 95% confidence level, whether less than 60% of students will attend. Denote p to be the true percentage of students who will show up. We have

$$H_0 : p = 0.6$$
$$H_a : p < 0.6$$

(a) What is the type I error here? What is the potential consequence for type I error?

(b) What is the type II error here? What is the potential consequence for type II error?

Solution:

(a) P(type I error) = P(We conclude $p < 0.6$, but p=0.6 is true).
The decision to reject H_0 but in reality H_0 is true, which means we believe less than 60% of the students will attend, but in reality, 60% will show up in that party night. There might not be enough refreshments or chairs to serve as potential consequence.

(b) P(type II error) = P(We conclude p = 0.6 but p < 0.6 is true).
We decide to accept H_0 but in reality H_0 is not true, which means we believe 60% of the students will attend, but in reality, less than 60% will show up in that party night. You would waste money on preparing large amount of refreshments as potential consequence.