Homework 13 Solution

STAT 509 Statistics for Engineers
Summer 2017 Section 001
Instructor: Tahmidul Islam

1. According to research published in Science (Feb 20,2004), the mere belief that you are receiving an effective treatment for pain can reduce the pain you actually feel. Researchers tested the placebo effect on 24 volunteers. Each volunteer was put inside an MRI for two consecutive sessions. During the first session electric shocks were applied to their arms and the blood oxygen level-dependent (BOLD) signal (a measure related to neural activity in the brain) was recorded during pain. The second session was identical to the first, except that prior to applying the electric shocks the researchers smeared a cream on the volunteer's arms. The volunteers were informed that the cream would block the pain, when, in fact, it was just a regular skin lotion (ie, placebo). If the placebo is effective in reducing pain, the BOLD measurements should be higher on average, in the first MRI session than in the second MRI session. The differences are calculated by subtracting second MRI measurement from first MRI measurement.

   (a) State the null and alternative hypotheses. (*Hint: this is a dependent 2-sample problem.*)

$$H_0 : \mu_D = 0$$
$$H_a : \mu_D > 0$$

   (b) The differences between the first BOLD measurements and the second were computed and the summarized results is as follows:

Variable	n	$\overline{y}_D$	s_D
size	24	0.21	0.47

   Calculate the test statistic.

$$\begin{aligned} t_0 &= \frac{\bar{y}_D - 0}{s_D/\sqrt{n}} \\ &= \frac{0.21 - 0}{0.47/\sqrt{24}} \\ &= 2.1889 \end{aligned}$$

   (c) Calculate the p-value.

$$P(T > 2.1889) = \texttt{1 - pt(2.1889, 24-1)} = 0.02$$

   (d) Make Decision and state your conclusion with 95% confidence.
   Since the p-value $= 0.02 < 0.05 = \alpha$, we reject the null and conclude that we have sufficient evidence that the BOLD measurements is higher on average without using the placebo, which means the argument that the belief you are receiving an effective treatment for pain can reduce the pain you actually feel is true.

2. A programmable lighting control system is being designed. The purpose of the system is to reduce electricity consumption costs in buildings. The system eventually will entail the use of a large number of transceivers (a device comprised of both a transmitter and a receiver). Two types of transceivers are being considered. In life testing, 200 transceivers (randomly

selected) were tested for each type. From transceiver 1, 20 failures were observed (out of 200), and from transceiver 2, 14 failures were observed (out of 200). The engineers want to test for the equality of the proportions. Define p_1 (p_2) to be the population proportion of transceiver 1 (transceiver 2) failures.

(a) State the null and alternative hypotheses.

$$H_0 : p_1 - p_2 = 0$$
$$H_a : p_1 - p_2 \neq 0$$

(b) Calculate the test statistic.
From the data, we have

$$\hat{p}_1 = \frac{20}{200} = 0.1 \qquad \hat{p}_2 = \frac{14}{200} = 0.07 \qquad \hat{p}_0 = \frac{20+14}{200+200} = 0.085$$

Thus,

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_0(1-\hat{p}_0)(\frac{1}{n_1}+\frac{1}{n_2})}} = \frac{0.1 - 0.07}{\sqrt{0.085(1-0.085)(\frac{1}{200}+\frac{1}{200})}} = 1.0757$$

(c) Calculate the p-value.

$$\text{p-value} = 2P(Z < -|z_0|) = 2P(Z < -1.0757) = 2 \times 0.142 = 0.282$$

(d) What is your decision and conclusion at $\alpha = 0.05$?
Since p-value is greater than α we conclude with confidence that there is no difference between the failure rate of trnsceiver 1 and transceiver 2.

(e) Use `prop.test` in `R` to check your work.

```
prop.test(c(20,14),c(200,200),correct=F)
```

The `prop.test()` gives us the similar p-value as we have calculated.

```
2-sample test for equality of proportions without continuity correction

data:  c(20, 14) out of c(200, 200)
X-squared = 1.1572, df = 1, p-value = 0.2821
alternative hypothesis: two.sided
95 percent confidence interval:
-0.02458069  0.08458069
sample estimates:
prop 1 prop 2
0.10   0.07
```