Homework 02 Solution

STAT 509 Statistics for Engineers
Summer 2017 Section 001
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Question 01

A and B are two events in the sample space S. Assume P(A) = 0.25 and P(B) = 0.4 and $P(A \cup B) = 0.6$. Calculate the following probabilities. Clearly state what probability rules you used. (Hint: You can draw Venn Diagrams to help you visualize the relationship among two events and the sample space.)

a $P(\bar{A})$.

b $P(A \cap B)$.

c $P(\bar{A} \cup \bar{B})$.

d $P(B|A)$.

e Are A and B are independent events? Why or why not?

Solution:

(a) $P(\bar{A}) = 1 - P(A) = 1 - 0.25 = 0.75$.

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

So $P(A \cap B) = P(A) + P(B) - P(A \cup B) = .25 + 0.4 - 0.6 = 0.05$.

(c) DeMorgan's Law: $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - 0.05 = 0.95$.

(d) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)} = \frac{0.05}{0.25}$.

(e) $P(A)P(B) = 0.25 \times 0.4 = 0.1 \neq P(A \cap B)$. So not independent.

Or,

$P(B|A) = 0.25 \neq P(B)$. So, not independent.

OR,

Find $P(A|B)$ and compare with $P(A)$.

Question 02

Transactions to a computer database are either new items or changes to previous items. The addition of an item can be completed in less than 100 milliseconds 90% of the time, but only 20% of changes to a previous item can be completed in less than this time. If 30% of transactions are changes, what is the probability that a transaction can be completed in less than 100 milliseconds? (Try to draw a tree diagram).

Solution:

$P(Change) = 0.30$; $P(New) = 0.70$.

$P(T < 100|New) = 0.90$, $P(T < 100|Change) = 0.20$.

Using law of total probability,

$P(T < 100) = P(T < 100|Change)P(Change) + P(T < 100|New)P(New)$

$= (.2 \times .3) + (.9 \times .7) = 0.69$.

Question 03

The probability that a randomly chosen automobile will need an oil change is 0.25; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.14.

a What is the probability that a car will need an oil change or new filter?

b If the oil had to be changed, what is the probability that a new oil filter is needed?

c If a new oil filter is needed, what is the probability that the oil has to be changed?

d Are oil filter and needing an oil change independent of one another. Why?

Solution:

(a)

$$\begin{aligned} P(Oil \cup Filter) =& P(Oil) + P(Filter) - P(Oil \cap Filter) \\ =& 0.25 + 0.40 - 0.14 = 0.51 \end{aligned}$$

(b)

$$\begin{aligned} P(Filter|Oil) =& \frac{P(Filter \cap Oil)}{P(Oil)} \\ =& \frac{0.14}{0.25} = 0.56 \end{aligned}$$

(c)

$$\begin{aligned} P(Oil|Filter) =& \frac{P(Filter \cap Oil)}{P(Filter)} \\ =& \frac{0.14}{0.40} = 0.35. \end{aligned}$$

(d)

$P(Filter|Oil) \neq P(Filter)$. So not independent.
Or
$P(Oil|Filter) \neq P(Oil)$. So not independent.
Or
$P(Oil) \times P(Filter) = 0.25 \times 0.40 = 0.1 \neq P(Filter \cap Oil)$. So not independent.