Homework 03

STAT 509 Statistics for Engineers
Summer 2017 Section 001
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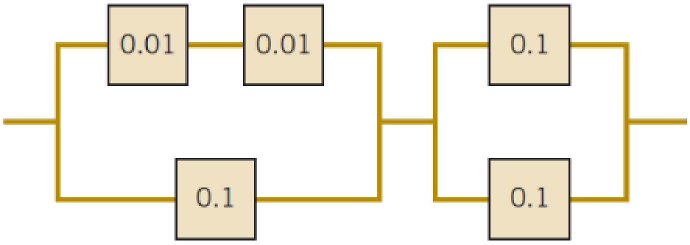
Question 01

The following circuit operates if and only if there is a path of functional devices from left to right. Assume devices fail independently and that the probability of failure of each device is as shown. What is the probability that the circuit operates?



Solution:

Lets compute the probability of functionality of the upper portion of the first stage.
P(both of the components functional) $= 0.99 \times 0.99 = 0.9801$. (Using independence)
Now, P(first stage functional) = P(upper or lower part functional)
= 1 - P(upper and lower not functional)
= 1 - P(upper part not functional) $\times$ P(lower part not functional); (Using independence)
$= 1 - (1 - 0.9801) \times 0.1$
$= 0.99801$.

Now we compute the probability of functionality of the second stage.
P(second stage functional) = P(1st or 2nd component working)
= 1 - P(both components failed) $= 1 - (.1 \times .1) = 0.99$.

Now P(the whole circuit functional) = P(first stage functional) $\times$ P(second stage functional)
$= 0.99801 \times 0.99 = 0.9880299$.

Question 02

A computer system uses passwords that contain exactly six characters, and each character is one of the 26 lowercase letters (a-z) or 26 uppercase letters (A-Z) or 10 integers (0-9). Let S denote the sample space of all possible password, and let A and B denote the events that consist of passwords with only letters or only integers, respectively. Suppose that all passwords in S are equally likely. Determine the following probabilities:

a $P(A)$.

b P(password contains at least 1 integer).

c P(password contains exactly 2 integers given that it contains at least 1 integer).

Solution:

(a) Total possible options available $= 26 + 26 + 10 = 62$.
Total number of possible passwords $= 62^6$.
Total number of possible passwords consisting only letters $= 52^6$.

$P(A) = \frac{52^6}{62^6} = (\frac{52}{62})^6 = 0.3480727.$

(b) P(password contains at least 1 integer) = 1 - P(password contains no integer) = 1 - P(A) = 0.7608026.

(c)

$$
\begin{aligned}
&P(\text{password contains exactly 2 integers given that it contains at least 1 integer}) \\
&= \frac{P(\text{password contains exactly 2 integers} \cap \text{it contains at least 1 integer})}{P(\text{it contains at least 1 integer})} \\
&= \frac{P(\text{password contains exactly 2 integers})}{P(\text{it contains at least 1 integer})} \\
&P(\text{password contains exactly 2 integers}) = \frac{\binom{6}{2} \times 10^2 \times 52^4}{62^6} \\
&= \binom{6}{2}(\frac{52}{62})^4 \times (\frac{10}{62})^2 = 0.1930876. \\
&P(\text{password contains exactly 2 integers given that it contains at least 1 integer}) \\
&= 0.1930876/0.7608026 = 0.2537946
\end{aligned}
$$

Question 03

Best Buy gives a choice of 3 CPU models, 2 monitors, 3 printers and 2 scanners. They can operate in any combination. In other words, any CPU can be used with any monitor which works with any printer, etc.

a If a configuration contains 1 CPU, 1 monitor, 1 printer and 1 scanner, how many configurations are possible?

b What is the probability of choosing one specific configuration?

c If the scanner is optional, how many configurations are possible?

Solution:

(a) $3 \times 2 \times 3 \times 2 = 36$.

(b) $P(\text{choosing one specific configuration}) = \frac{1}{36}$.

(c) $(3 \times 2 \times 3 \times 2) + (3 \times 2 \times 3) = 54$.