

Homework 06

STAT 509 Statistics for Engineers

Summer 2017 Section 001

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Question 01

One day Shiwen is very hungry and Jeff gives Shiwen a big box of chocolates. When Shiwen starts to eat, Jeff says 5 percent of chocolates are made with Carolina Reaper. Shiwen is too hungry to stop eating.

- Let X denote the number of chocolates Shiwen eats until the first one made with Carolina Reaper. Find $P(X > 3)$. Interpret what this probability means in words.
- In part (a), calculate $P(X > 5|X > 2)$. Interpret what this probability means in words. (Hint: use conditional probability formula).
- Compare $P(X > 3)$ and $P(X > 5|X > 2)$, which one is larger? Can you explain the reason?
- Suppose on that day Shiwen eats 30 pieces of chocolates. Find the probability that no more than two of those chocolates are made with Carolina Reaper.

Question 02

A recent geological study in western Texas indicates that exploratory oil wells strike with probability 0.20. (i.e., oil is found).

- Treating each well as a “trial”, suppose that drilling wells in this region obeys the three Bernoulli trial assumptions. State what this would imply (i.e., describe the three assumptions in the background of the problem).
- What is the probability that the 1st successful well is found on the 2th well drilled?
- What is the probability that the 2th successful well is found on the 4th well drilled?
- What is the probability that it will take more than 2 wells to find the 1nd successful well?

Question 03

Jeff gives Shiwen a box of 100 pieces of chocolate, in which 5 are made with Carolina Reaper.

- Shiwen randomly eats 10 pieces of chocolate, what is the probability that none of them is made with Carolina Reaper?
- Shiwen randomly eats 10 pieces of chocolate, what is the probability that at most 2 pieces of chocolates are made with Carolina Reaper?

Question 04

Let Y be the number of calls received per day by the USC Campus Police. Suppose that Y has a Poisson distribution with $\lambda = 6.5$.

- What is the probability that on a given day there are exactly 5 calls? at least 5 calls? at most 5 calls?
- What is the mean of Y ? Interpret it using the context of the problem.

c Suppose that the daily cost (in dollars) to respond to Y calls is given by

$$g(Y) = 150 + 100Y + 5Y^2.$$

Find the expected daily cost. (Hint: $\text{var}(Y) = E(Y^2) - [E(Y)]^2$).

d Suppose that in a given week, there are 30 calls received. Twelve of the calls involved illegal consumption of alcohol and 18 did not. If administration picks 5 cases (calls) to review at random, what is the probability that at least 4 of these cases will involve illegal consumption of alcohol?