Homework 07 Solution

STAT 509 Statistics for Engineers
Summer 2017 Section 001
Instructor: Tahmidul Islam

Question 01

In 1990 the lead concentration in gasoline ranged from 0.1 to 0.5 grams/liter. Let Y = grams per liter of lead in gasoline. The probability density function for Y is

$$f(y) = 12.5y - 1.25;\ 0.1 < y < 0.5.$$

(a) What is the probability that a random liter of gasoline would contain between 0.1 and 0.4 grams/liter of lead?

(b) What is the probability that a random liter of gasoline will contain more than 0.3 grams/liter of lead?

(c) Give the cumulative probability function $F_Y(y)$. (Hint: you need to discuss the value of y for three cases: $y \leq 0.1, 0.1 < y < 0.5, Y \geq 0.5$).

(d) Use the cumulative probability function $F_Y(y)$ to calculate the probability that a random liter of gasoline will contain less than 0.35 grams of lead.

(e) Calculate the expected value of Y.

(f) Calculate the variance for Y.

Solution:

(a)

$$\begin{aligned}
P(Y < 0.4) &= \int_{0.1}^{0.4} f(y)dy \\
&= \int_{0.1}^{0.4} (12.5y - 1.25)dy \\
&= [12.5\frac{y^2}{2} - 1.25y]_{0.1}^{0.4} \\
&= (\frac{12.5 \times 0.4^2}{2} - 1.25 \times 0.4) - (\frac{12.5 \times 0.1^2}{2} - 1.25 \times 0.1) \\
&= 0.5625.
\end{aligned}$$

(b)

$$\begin{aligned}
P(Y > 0.3) &= \int_{0.3}^{0.5} f(y)dy \\
&= \int_{0.3}^{0.5} (12.5y - 1.25)dy \\
&= [12.5\frac{y^2}{2} - 1.25y]_{0.3}^{0.5} \\
&= (\frac{12.5 \times 0.5^2}{2} - 1.25 \times 0.5) - (\frac{12.5 \times 0.3^2}{2} - 1.25 \times 0.3) \\
&= 0.75.
\end{aligned}$$

(c)

$$\begin{aligned}F_Y(y) =&P(Y \leq y)\\=&\int_{0.1}^{y}(12.5u - 1.25)du\\=&[12.5\frac{u^2}{2} - 1.25u]_{0.1}^{y}\\=&(12.5\frac{y^2}{2} - 1.25y) - (12.5\frac{.1^2}{2} - 1.25 \times .1)\\=&6.25y^2 - 1.25y + 0.0625.\end{aligned}$$

So the CDF can be written as

$$F_Y(y) = \begin{cases}0;\ y \leq 0.1\\6.25y^2 - 1.25y + 0.0625;\ 0.1 < y < 0.5\\1;\ y \geq 0.5.\end{cases}$$

(d)

$$\begin{aligned}P(Y < 0.35) =&F_Y(0.35)\\=&6.25 \times .35^2 - 1.25 \times .35 + 0.0625\\=&0.390625\end{aligned}$$

(e)

$$\begin{aligned}E(Y) =& \int_{0.1}^{0.5} yf(y)dy\\=&\int_{0.1}^{0.5} y(12.5y - 1.25)dy\\=&[12.5\frac{y^3}{3} - 1.25\frac{y^2}{2}]_{0.1}^{0.5}\\=&(12.5\frac{.5^3}{3} - 1.25\frac{.5^2}{2}) - (12.5\frac{.1^3}{3} - 1.25\frac{.1^2}{2})\\=&0.3666666.\end{aligned}$$

Question 02

Suppose the weight, say, Y, in pounds of a certain packaged chemical is uniform from 48 to 50 pounds. That is the pdf is of the form

$$f_Y(y) = \frac{1}{2};\ 48 \leq y \leq 50.$$

(a) What is the mean weight of the chemical?

(b) What is the probability that a randomly chosen package of chemical will weigh between 48.5 and 49.4 pounds?

(c) In the long run, what proportion of packages will weigh more than 49.2 pounds?

Solution:

(a)

$$\begin{aligned}E(Y) =& \int_{48}^{50} y\ 0.5\ dy \\ =& 0.5 \int_{48}^{50} y dy = 0.5[\frac{y^2}{2}]_{48}^{50} \\ =& \frac{1}{4}(50^2 - 48^2) \\ =& 49.\end{aligned}$$

Or you can informally arrive at this by observing that 49 is the middle point between 48 and 50 and the distribution is uniform.

(b) $P(48.5 < Y < 49.4) = (49.4 - 48.5) \times 0.5 = 0.45$. (Draw the pdf, identify the interval, then picture area under the curve, here curve is just a line, so area under the curve is just area of a rectangle). This can also be found using the definition $P(48.5 < Y < 49.4) = \int_{48.5}^{49.4} f_Y(y)dy$.

(c) $P(Y > 49.2) = (50 - 49.2) \times 0.5 = 0.4$. (Using similar technique like above).