Hypothesis Testing for Proportion
Confidence Intervals to Testing

• As we see in chapter 8.1 and 8.2 we can come up with interesting observations given our confidence intervals

• Next we will learn how to formally test whether or not the population proportion is a particular value based on our sample proportion
Hypothesis

• A **Hypothesis** is a proposition assumed as a premise in an argument. It’s a statement regarding a characteristic of one or more populations.

• **Hypothesis testing** is a procedure based on evidence found in a sample to test hypothesis
Hypothesis

• The **null hypothesis**, $H_0$, is a statement to be tested. The null hypothesis is a statement of no change, no effect or no difference and is assumed true until evidence indicates otherwise.

• The **alternative hypothesis**, $H_1$ or $H_a$, is a statement that we are trying to find evidence to support.
Hypothesis

1. Two-tailed test
   • $H_0$: parameter = some value
   • $H_a$: parameter ≠ some value

2. Left-tailed test
   • $H_0$: parameter ≥ some value
   • $H_a$: parameter < some value

3. Right-tailed test
   • $H_0$: parameter ≤ some value
   • $H_a$: parameter > some value
Hypothesis Test for Proportions: Step 1

• We are interested in testing whether the population proportion, \( p \), is equal, or great, or less than \( p_o \).

• Step 1 is to know what hypothesis you want to test.

<table>
<thead>
<tr>
<th>Two-tailed test</th>
<th>Left-tailed test</th>
<th>Right-tailed test</th>
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</thead>
<tbody>
<tr>
<td>( H_0: p = p_o )</td>
<td>( H_0: p \geq p_o )</td>
<td>( H_0: p \leq p_o )</td>
</tr>
<tr>
<td>( H_a: p \neq p_o )</td>
<td>( H_a: p &lt; p_o )</td>
<td>( H_a: p &gt; p_o )</td>
</tr>
</tbody>
</table>
Hypothesis Test for Proportions: Step 2

• Check the assumptions:
  1. The variable must be categorical
  2. The data should be obtained using randomization
  3. The sample size is sufficiently large where $p_o$ is the testing value satisfying
     • $np_o \geq 15$
     • $n(1 - p_o) \geq 15$
     • It is safe to assume the distribution of $p_o$ has a bell shaped distribution
Hypothesis Test for Proportions: Step 3

• Calculate Test Statistic, $z^*$
  • The test statistic measures how different the sample proportion we have is from the null hypothesis
  • We calculate the $z$-score by assuming that $p_o$ is the population proportion

\[
z^* = \frac{(\hat{p} - p_o)}{\sqrt{p_o(1 - p_o) \frac{1}{n}}}\]
Hypothesis Test for Proportions: Step 4

• Determine the P-value
  • What is P-value?
  • The P-value describes how unusual/unlikely the sample data would be if $H_0$ were true.
  • $z^*$ is the test statistic from step 3

<table>
<thead>
<tr>
<th>Alternative Hypothesis</th>
<th>Probability</th>
<th>Formula for the P-value</th>
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<tbody>
<tr>
<td>$H_a: p &gt; p_o$</td>
<td>Right tail</td>
<td>$P(Z&gt;z^*)$</td>
</tr>
<tr>
<td>$H_a: p &lt; p_o$</td>
<td>Left tail</td>
<td>$P(Z&lt;z^*)$</td>
</tr>
<tr>
<td>$H_a: p \neq p_o$</td>
<td>Two-tail</td>
<td>$2*P(Z&lt;-</td>
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</tbody>
</table>
Hypothesis Test for Proportions: Step 5

- Summarize the test by reporting and interpreting the P-value
  - Smaller p-values give stronger evidence against $H_0$
- If p-value $\leq (1 - confidence) = \alpha$
  - Reject $H_0$, with a p-value = ____, we have sufficient evidence that the alternative hypothesis might be true.
- If p-value $> (1 - confidence) = \alpha$
  - Fail to reject $H_0$, with a p-value = ____ , we do not have sufficient evidence that the alternative hypothesis might be true.
Hypothesis Test for Proportions: Step 5 (cont.)

• For a left tailed test: $H_a: p < p_0$ → We have rejection regions for $H_o$ are as follows
  • Note: all of the rejection region is in the left tail, where $\hat{p}$ is much smaller than $p_0$

<table>
<thead>
<tr>
<th>Confidence</th>
<th>Reject (test stat)</th>
<th>Reject (p-value)</th>
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<tbody>
<tr>
<td>0.90</td>
<td>Test-stat&lt;-1.282</td>
<td>P-value&lt;.1</td>
</tr>
<tr>
<td>0.95</td>
<td>Test-stat&lt;-1.645</td>
<td>P-value&lt;.05</td>
</tr>
<tr>
<td>0.99</td>
<td>Test-stat&lt;-2.326</td>
<td>P-value&lt;.01</td>
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Hypothesis Test for Proportions: Step 5 (cont.)

- For a right tailed test: $H_a: p > p_0$ → We have rejection regions for $H_0$ are as follows
  - Note: all of the rejection region is in the right tail, where $\hat{p}$ is much larger than $p_0$

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</tr>
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<td>0.99</td>
<td>Test-stat&gt;2.326</td>
<td>P-value&lt;.01</td>
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Confidence | Reject (test stat) | Reject (p-value)
--- | --- | ---
0.90 | Test-stat>1.282 | P-value<.1
0.95 | Test-stat>1.645 | P-value<.05
0.99 | Test-stat>2.326 | P-value<.01
Hypothesis Test for Proportions: Step 5 (cont.)

• For a two tailed test: $H_a: p \neq p_0 \rightarrow$ We have rejection regions for $H_o$ are as follows
  • Note: here we split the rejection region into both tails, where $\hat{p}$ is very different from $p_0$

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![Diagram of normal distribution with rejection regions](image_url)
## Zoom In

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Example: Jar Jar Binks
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• 340 randomly selected people were asked whether or not they liked Jar Jar Binks
• 23 people said that they did like
• At the 0.05 level of significance, or 95% confidence, is there evidence that less than 10% of all people like Jar Jar Binks?

• $\hat{p} = \frac{23}{340} = 0.068$
Example: Step 1

• State the Hypothesis: we are interested in whether or not less than ten percent of all people like Jar Jar Binks
  ▪ $H_o : p \geq 0.10$
  ▪ $H_a : p < 0.10$
Example: Step 2

• Check Assumptions
  • The variable is categorical
    • They like Jar Jar Binks or they don’t like
  • The data was collected randomly
• $np_o = 340 \times (0.1) = 34 \geq 15$
• $n(1 - p_o) = 340 \times (0.9) = 306 \geq 15$
Example: Step 3

• Calculate the test statistic:

\[ z^* = \frac{(\hat{p} - p_0)}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{(0.068 - 0.1)}{\sqrt{\frac{0.1(1-0.1)}{340}}} = -1.99 \]
Example: Step 4

• Determine P-value for left tailed test
  • From the table $p - value = P(Z < z^*)$

$$P(Z < -1.99) = .0233$$
Example: Step 5

• State Conclusion
  • Since $0.0233 < 0.05$ we reject $H_o$

At the 0.05 level of significance, or 95% confidence level, there is sufficient evidence that fewer than 10% of all people like Jar Jar Binks.
Example: Step 5 with Picture

• State Conclusion
  • Anything with a p-value<0.05 or a z-value<1.645 will be in the rejection region
  • Since p-value=0.0233 < α = 1 − 0.95 = 0.05 we reject $H_o$
Example: Step 5 with different $\alpha$

• **Note:** this would not be the case if we tested at the .01 level of significance, or 99% confidence level.

• **State Conclusion**
  • Since $0.0233 > 1 - 0.99 = 0.01$ we fail to reject $H_0$
  
  At the 0.01 level of significance, or 99% confidence level, there is not sufficient evidence that fewer than 10% of all people like Jar Jar Binks
Example: Step 5 with different $\alpha$

- State Conclusion
  - Anything with a p-value $< 0.01$ or a z-value $< 2.326$ will be in the rejection region
  - Since $p$-value $= .0233 < \alpha = 1 - .99 = .01$ we fail to reject $H_0$
Example: Hogwarts
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• 1002 randomly selected students were asked whether or not they would go to Hogwarts if admitted
• 701 said they would
• At the 0.05 level of significance (95% confidence) is there evidence that the proportion of students who say they would go to Hogwarts differs from 0.69
• $\hat{p} = \frac{701}{1002} = 0.6996$
Example: Step 1

• State the Hypothesis: we are interested in whether or not the population proportion of people that would go to Hogwarts if accepted is different from 0.69
  • $H_o: p = 0.69$
  • $H_a: p \neq 0.69$
Example: Step 2

• Check Assumptions
  ▪ The variable is categorical
    • They either would go or they wouldn’t
    • The data was collected randomly
  ▪ \( np_o = 1002 \times 0.69 = 691.38 \geq 15 \)
  ▪ \( n(1 - p_o) = 1002 \times 0.31 = 310.62 \geq 15 \)
  ▪ So, it is safe to assume the distribution of \( p_o \) has a bell shaped distribution
Example: Step 3

• Calculate the test statistic:

\[ z^* = \frac{(\hat{p} - p_o)}{\sqrt{p_o(1 - p_o)/n}} = \frac{(0.6996 - 0.69)}{\sqrt{0.69(1 - 0.69)/1002}} = 0.6571 \approx 0.66 \]
Example: Step 4

• Determine P-value for left tailed test
  • From the table $p - value = 2 \times P(Z < -|z^*|)$

\[
2 \times P(Z < -|0.66|) = 2 \times P(Z < -0.66) = 2 \times 0.2545 = 0.5090
\]
Example: Step 5

• State Conclusion
  • Since $0.5090 > 0.05$ we fail to reject $H_o$

At the 0.05 level of significance, or 95% confidence level, there is not sufficient evidence that the proportion of people that would go to Hogwarts if accepted differs from 0.69
Example: Step 5 with Picture

• State Conclusion

  • Anything with a p-value < 0.025 or a z-value < -1.96 or z-value > 1.96 will be in the rejection region
  • Since p-value = 0.5090 < α = 1 - 0.95 = 0.05 we fail to reject $H_0$
Example: MLB

- Do you remember the sad story in chapter 8 about Jerry teases your boss, Tom, and you got fired?
- A random sample of MLB home games showed that the home teams won 1335 of 2429 games.
- At the 0.01 level of significance (99% confidence) is there evidence that there is a home field advantage?

\[ \hat{p} = \frac{1335}{2429} = 0.5496 \]
Example: Step 1

• State the Hypotheses: we are interested in whether or not there was a home field advantage, which is whether or not the population proportion of home games won by the home team is greater than .50

• $H_0: p \leq 0.5$
• $H_a: p > 0.5$
Example: Step 2

- **Check Assumptions**
  - The variable is categorical
    - Either the home team won or they didn’t
  - The data was collected randomly
  - \( n p_o = 2429 \times 0.5 = 1214.5 \geq 15 \)
  - \( n(1 - p_o) = 2429 \times 0.5 = 1214.5 \geq 15 \)
  - So, it is safe to assume the distribution of \( p_o \) has a bell shaped distribution
Example: Step 3

• Calculate the test statistic:

\[ z^* = \frac{(\hat{p} - p_o)}{\sqrt{\frac{p_o(1 - p_o)}{n}}} = \frac{(0.5496 - 0.5)}{\sqrt{\frac{0.5(1 - 0.5)}{2429}}} = 4.89 \]
Example: Step 4

• Determine P-value
  • From the table $p-value = P(Z > z *)$

\[
p-value = P(Z > 4.89) = 1 - P(Z < 4.89) \\
\approx 1 - 1 = 0
\]
Example: Step 5

• State Conclusion
  • Since $0 < 0.01$ we reject $H_0$

At the 0.01 level of significance, or 99% confidence level, there is sufficient evidence to suggest that there is a home field advantage
Confidence Interval VS. Hypothesis Testing

• In Chapter 8, we have 99% confidence interval (0.523, 0.575)
• In chapter 9, we make a rejection of null with 99% confidence when asking hypothesis
  • $H_o: p \leq 0.5$
  • $H_a: p > 0.5$
• Can you see the link between the confidence interval and the hypothesis testing?