L15-2: Sections 9.2 and 9.4

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Elementary Statistics for the Biological and Life Sciences

Dichotomous observations

- In Chapters 5 (sampling distribution), 6 (confidence interval), 7 (two independent samples), and 8 (paired samples) we have dealt with continuous observations Y.
- With continuous observations it is natural to look at the mean or median.
- Now we'll consider categorical data.
- The simplest categorical data simply tells us which of two categories a subject is in, e.g. Male or Female, Diseased or Non-diseased, ≥ 50 years or < 50 years, etc.
- This type of data is called *dichotomous* or *binary*.
- For categorical data, we'll concentrate on the proportion of one category (successes) p.

Flip a coin 100 times

Head	Tail	Total
43	57	100

Checking whether a coin is fair.
Label head as “success” and testing the proportion of successes.

Example 9.4.9 Harvest Moon Festival

- Can people close to death postpone dying until after a meaningful event?
- Researchers studied death from natural causes among Chinese women over age 75 living in California. Traditionally, during Harvest Moon oldest woman in the family has important role.
- Previous research had suggested that there might be a decrease in the mortality rate among elderly Chinese women immediately prior to the festival, with a corresponding increase afterward.

Example 9.4.9 Harvest Moon Festival

- The researchers found that over a period of several years there were 33 deaths in the group in the week preceding the Harvest Moon Festival and 70 deaths in the week following the festival.
- If people cannot postpone, then probability of dying before is same as dying after, 50%.

Table 9.4.4 Harvest moon festival data

	Before	After	Total
Observed	33	70	103
Expected	51.5	51.5	103

Test of proportions

- Pick one of the two categories to be "success." Then we have a population with an unknown proportion p of success.
- We often want to test $H_0 : p = p_0$ where p_0 is a known value.
- If we take a random sample on n individuals and count Y, the number of "successes" then $Y \sim \text{binom}(n, p_0)$ if $H_0 : p = p_0$ is true.
- The sample proportion $\hat{p} = Y/n$ estimates the true population proportion. If $\hat{p}$ is far away from p_0 then we have evidence of $p \neq p_0$.

P-value for Harvest Moon Festival data

- Let p be proportion that die before Harvest moon. Suppose we want to test $H_0 : p = 0.5$ vs. $H_A : p < 0.5$.
- Here $\hat{p} = \frac{33}{103} = 0.32$, smaller than the hypothesized value 0.5, so there's some evidence toward H_A.
- The P-value is computed

$$\text{P-value} = \Pr\{Y/n \leq \hat{p} | p = p_0\} = \Pr\{Y \leq 33 | p = 0.5\},$$

  the probability of seeing a $\hat{p}$ even less than 0.32, given H_0 is true.
- Since $Y \sim \text{binom}(103, 0.5)$ under H_0, we can find

$$\text{P-value} = \sum_{j=0}^{33} \Pr\{Y = j\} = 0.0001705 = \text{pbinom(33,103,0.5)}.$$

P-value from binom.test

R's function `binom.test` computes P-values for tests of $H_0 : p = p_0$ vs. one of (a) $H_A : p \neq p_0$, (b) $H_A : p < p_0$, or (c) $H_A : p > p_0$.

```
> binom.test(33,103,p=0.5,alternative="less")

        Exact binomial test

data:  33 and 103
number of successes = 33, number of trials = 103, p-value = 0.0001705
alternative hypothesis: true probability of success is less than 0.5
95 percent confidence interval:
 0.0000000 0.4041263
sample estimates:
probability of success
             0.3203883
```

Since P-value $= 0.00017 < 0.05$ we reject $H_0 : p = 0.5$ in favor of $H_A : p < 0.5$. There is statistically significant evidence that people can postpone dying.

9.2 Confidence intervals

- The text book discusses one method for obtaining approximate confidence intervals for an unknown p; a somewhat cruder.
- `binom.test(y,n)` uses the relationship between confidence intervals and hypothesis tests to compute an exact confidence interval; other confidence intervals (including your book's) are approximate.

```
> binom.test(33,103)
95 percent confidence interval:
 0.2318410 0.4195741
```

We are 95% confident that the true probability of dying right before Harvest Moon (vs. right after) is between 23% and 42%. There is evidence that people "wait to die."

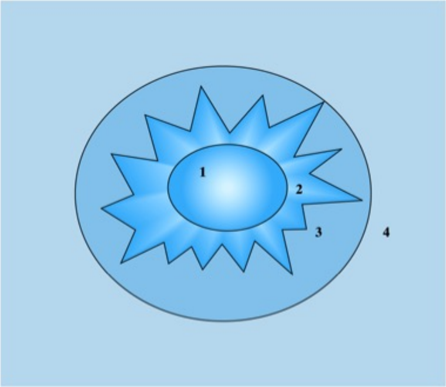
9.4 χ^2 goodness-of-fit test

- So far we have considered binary variables with unknown population $p = \Pr\{\text{success}\}$.
- Specifically, we tested $H_0 : p = p_0$ where p_0 is known.
- This is now generalized to categorical variables with more than two categories.
- Example: roll a dice

1	2	3	4	5	6	Total
98	102	85	115	82	118	600

- Example 9.4.1 Deer habitat and fire. Does fire affect deer behavior? After a fire burned 730 acres, researchers surveyed the 3,000 acres surrounding the area, divided into four regions.

Example 9.4.1 Deer Habitat and Fire



(1) the region near the heat of the burn, (2) the inside edge of the burn, (3) the outside edge of the burn, (4) the area outside of the burned area.

Null and alternative hypotheses

- H_0 : deer show no preference to any burned/unburned habitat, i.e. they are randomly distributed over the 3,000 acres.
- H_A : deer have a preference for some of the regions.
- Under the null hypothesis, the probabilities of deer in the regions are proportional to the sizes of the regions.

Table 9.4.1 Deer distribution

Region	Acres	Proportion
1. Inner burn	520	0.173
2. Inner edge	210	0.070
3. Outer edge	240	0.080
4. Outer unburned	2,030	0.677
	3,000	1.000

Observed vs. expected

- The researchers found 75 deer in the whole region. Before we find out how they were distributed across the 4 regions, what do we *expect* to see?
- If 17.3% of the deer are in the inner burn, then we expect $0.173(75) = 13.00$ of the 75 to be found there.
- We expect $0.070(75) = 5.25$ of the 75 in the inner edge.
- We expect $0.080(75) = 6.00$ of the 75 in the outer edge.
- We expect $0.677(75) = 50.75$ of the 75 in the outer unburned.
- The numbers *actually observed* are 2, 12, 18, and 43.

Observed vs. expected

Region	H_0 proportion	Observed proportion	Expected under H_0	Observed number
Inner burn	0.173	0.027	13.00	2
Inner edge	0.070	0.16	5.25	12
Outer edge	0.080	0.24	6.00	18
Outer unburned	0.677	0.57	50.75	43

Do you observe discrepancies between the expected vs observed deer counts?

Chi-square statistic

Let e_i be the expected number in each category and let o_i be the observed number. The chi-square goodness of fit test statistic is

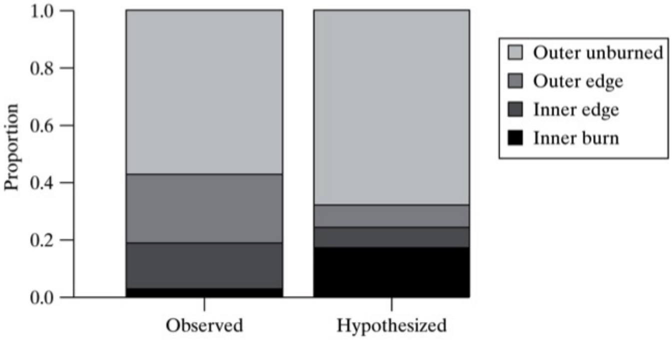
$$\chi_S^2 = \sum_{i=1}^{k} \frac{(o_i - e_i)^2}{e_i}$$

where the summation is over all k categories. Goodness-of-fit test.

For the deer data

$$\chi_S^2 = \frac{(2-13)^2}{13} + \frac{(12-5.25)^2}{5.25} + \frac{(18-6)^2}{6} + \frac{(43-50.75)^2}{50.75} = 43.2.$$

Deer proportions



If the gray-shaded areas have markedly different sizes across hypothesized and observed, then the χ^2_S will be large. If they are perfectly equal then $\chi^2_S = 0$.

χ^2 distribution

- If H_0 is true then χ^2_S has a special distribution called the chi-square distribution (p. 372).
- Like the t distribution, the χ^2 has degrees of freedom *df* attached to it; the $df = k - 1$, the number of categories minus one.
- The P-value for testing H_0 is the tail probability P-value $= \Pr\{\chi^2_{df} > \chi^2_S\}$.
- R takes care of details for us with the `chisq.test(counts,p=probabilities)` command.
- `counts` is a list of the observed numbers falling into each of the k categories and `probabilities` are the hypothesized probabilities under H_0.
- P-value is probability of seeing observed *even further away from expected* than we saw under H_0.

R command chisq.test

Need to define two lists: (1) the observed counts in each category, and (2) a list of the hypothesized H_0 probabilities that add up to one.

```
> deer=c(2,12,18,43)
> prob=c(0.173,0.070,0.080,0.677)
> chisq.test(deer,p=prob)
        Chi-squared test for given probabilities

data:  deer
X-squared = 43.1524, df = 3, p-value = 2.284e-09
```

Since P-value $= 0.0000000023 < 0.05$, we reject H_0 at the 5% level. There is statistically significant evidence that the deer prefer some areas to others, i.e. they are not randomly scattered around the burn area. The data tell us that deer prefer the inner and outer edges to the inner burn and outer unburned areas.