L19: Chapter 10: Contingency tables III

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Stat 205: Elementary Statistics for the Biological and Life Sciences

$2 \times 2 \times k$ tables

- So far we've considered one 2×2 table.
- The sample odds ratio $\hat{\theta}$ estimates the association between success and group membership.
- There may be another grouping variable that affects success, often called *strata*.
- If the additional grouping variable has k levels, we have a $2 \times 2 \times k$ table.

Example where $k = 3$

Stratified tables:

Strata	Outcome	Group A	Group B
Strata 1	Success	y_{11}	y_{12}
	Failure	$n_{11} - y_{11}$	$n_{12} - y_{12}$
Strata 2	Success	y_{21}	y_{22}
	Failure	$n_{21} - y_{21}$	$n_{22} - y_{22}$
Strata 3	Success	y_{31}	y_{32}
	Failure	$n_{31} - y_{31}$	$n_{32} - y_{32}$

Collapsed table ignoring strata:

Outcome	Group A	Group B
Success	$y_{11} + y_{21} + y_{31}$	$y_{12} + y_{22} + y_{32}$
Failure	$n_{11} + n_{21} + n_{31}$ $-y_{11} - y_{21} - y_{31}$	$n_{12} + n_{22} + n_{32}$ $-y_{12} - y_{22} - y_{32}$

Simpson's paradox

- **Paradox**: a self-contradictory and false proposition.
- If we ignore the additional strata variable, and estimate the odds ratio θ *from the collapsed table* the usual way by $\hat{\theta}$, we are omitting important information.
- Often the information obtained from the collapsed table *contradicts* information in the stratified tables.
- For example, the odds ratios from each of the k stratified tables $\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k$ can go in the opposite direction from the $\hat{\theta}$ from the collapsed table.
- Let's look at two examples...

Prenatal care and infant survival

Consider some data from the Harvard School of Public Health on infant survival. Each women in the study received prenatal care classified as "less" or "more." Does the amount of prenatal care relate to infant survival?

Outcome	Less care	More care
Died	20	6
Survived	373	316

- $\hat{\theta} = 2.82\ (1.08, 8.69)$, P-value $= 0.026$ using Fisher's exact test.
- Strong, positive association between increased prenatal care and survival.

Addition of strata "clinic"

Actually, each mother was treated in two clinics, A and B. The table on the previous slide was obtained by adding the tables for clinics A and B.

Location	Outcome	Less care	More care
Clinic A	Died	3	4
	Survived	176	293
Clinic B	Died	17	2
	Survived	197	23

- $\hat{\theta}_A = 1.25\ (0.18, 7.47)$, P-value $= 1$, $\hat{\theta}_B = 0.99\ (0.21, 9.41)$, P-value $= 1$.
- Within each clinic, there is no significant evidence of association between prenatal care and survival.
- Infants are way more likely to die at clinic B than A; this ends up driving what happens in the collapsed table.
- When we *stratify*, or adjust for a third variable, association can vanish or go in the opposite direction (Simpson's paradox).

Treatment for kidney stones

Charig et al. (1986) compare the success rates of two treatments for kidney stones. Treatment A includes all open procedures and Treatment B is percutaneous nephrolithotomy.

Outcome	Treatment A	Treatment B
Success	273	289
Failure	77	61

- $\hat{\theta} = 273 \times 61/(289 \times 77) = 0.75\ (0.50, 1.11)$.
  P-value $= 0.154$.
- Association is non-significant. But the point estimation of the odds ratio indicates that treatment B is better ($\hat{\theta} < 1$).

Table stratified by large or small stones

Stones size	Outcome	Treatment A	Treatment B
Small	Success	81	234
	Failure	6	36
Large	Success	192	55
	Failure	71	25

- $\hat{\theta}_S = 2.08$ and $\hat{\theta}_L = 1.23$. Treatment A is the better treatment when stratified by how big the stones are! Treatment B is better when ignoring kidney stone size.
- Success rate is influenced by two things: the treatment (A or B) *and how big the stones are*. In fact, success is more strongly influenced by by stone size than treatment A or B.
- Doctors tended to give the severe cases (large stones) the better treatment A, and the milder cases (small stones) treatment B; patients with large stones given treatment A do worse than those with small stones given B, and these two groups *dominate the collapsed table*.

Cochran-Mantel-Haenszel test

- The Cochran-Mantel-Haenszel test assumes each stratified table has the same odds ratio $\theta_1 = \theta_2 = \cdots = \theta_k = \theta_s$.
- θ_s (s is for "stratified") is the *common, conditional odds ratio*.
- θ is from the *collapsed table* – we don't want this one because of Simpson's paradox.
- The Cochran-Mantel-Haenszel test provides a P-value for $H_0 : \theta_s = 1$. If we reject then there is a significant association between success and group *within each stratification level*.
- It is also possible to let each strata have their own odds ratio and test $H_0 : \theta_1 = \theta_2 = \cdots = \theta_k = 1$. This can be done using *logistic regression*, but this is beyond the course.

Cochran-Mantel-Haenszel test in R

- The `mantelhaen.test(data)` function performs the Cochran-Mantel-Haenszel test on the contingency table stored in `data`.
- The contingency table is stored as a $2 \times 2 \times k$ array in R.
- Just like storing a 2×2 table as a matrix, we need to store a $2 \times 2 \times k$ table as an array.
- This is best illustrated with several examples.

Cochran-Mantel-Haenszel test for prenatal data

```
> prenatal=array(c(3,176,4,293,17,197,2,23),dim=c(2,2,2))
> prenatal
, , 1
     [,1] [,2]
[1,]    3    4
[2,]  176  293

, , 2
     [,1] [,2]
[1,]   17    2
[2,]  197   23

> mantelhaen.test(prenatal)

        Mantel-Haenszel chi-squared test without continuity correction

data:  prenatal
Mantel-Haenszel X-squared = 0.0386, df = 1, p-value = 0.8442
alternative hypothesis: true common odds ratio is not equal to 1
95 percent confidence interval:
 0.3759998 3.2977887
sample estimates:
common odds ratio
         1.113539
```

$\hat{\theta}_s = 1.11$ $(0.38, 3.30)$ w/ P-value$= 0.84$. No association.

Cochran-Mantel-Haenszel test for kidney stones

```
> stones=array(c(81,6,234,36,192,71,55,25),dim=c(2,2,2))
> stones
, , 1
     [,1] [,2]
[1,]   81  234
[2,]    6   36

, , 2
     [,1] [,2]
[1,]  192   55
[2,]   71   25

> mantelhaen.test(stones,alternative="greater")

        Mantel-Haenszel chi-squared test with continuity correction

data:  stones
Mantel-Haenszel X-squared = 2.0913, df = 1, p-value = 0.07407
alternative hypothesis: true common odds ratio is greater than 1
95 percent confidence interval:
 0.9856691       Inf
sample estimates:
common odds ratio
         1.446847
```

P-value$=0.07$ for testing $H_0 : \theta_s = 1$ vs. $H_A : \theta_s > 1$.

Elk behavior

- Gagnon et al. (2007) studied elk use of wildlife underpasses on a highway in Arizona.
- Using video surveillance cameras, they recorded each elk that started to cross under the highway. When a car or truck passed over while the elk was in the underpass, they recorded whether the elk continued through the underpass ("crossing") or turned around and left ("retreat").
- Overall traffic volume categorized as low (fewer than 4 vehicles per minute) or high.

Elk behavior data

- It is of interest to determine whether there's a relationship between behavior and type of vehicle, adjusting for traffic level.
- "Adjusting" is another word for "controlling for" – here "stratifying."
- Here's the data:

Location	Vehicle	Car	Truck
Low traffic	Crossing	287	40
	Retreat	57	42
High traffic	Crossing	237	57
	Retreat	52	12

Let's fit this in R...

R code for elk behavior data

```
> elk=array(c(287,57,40,42,237,52,57,12),dim=c(2,2,2))
, , 1
     [,1] [,2]
[1,]  287   40
[2,]   57   42

, , 2
     [,1] [,2]
[1,]  237   57
[2,]   52   12
> mantelhaen.test(elk)

        Mantel-Haenszel chi-squared test with continuity correction

data:  elk
Mantel-Haenszel X-squared = 24.39, df = 1, p-value = 7.868e-07
alternative hypothesis: true common odds ratio is not equal to 1
95 percent confidence interval:
 1.801123 3.924165
sample estimates:
common odds ratio
         2.658553
```

Interpretation

- $\hat{\theta}_s = 2.66$.
- The odds of crossing are estimated to be almost three times greater for cars than trucks, *adjusting for traffic level*.
- We are 95% confident that the true odds of crossing are between 2 and 4 times greater for cars, adjusting for traffic.
- Since P-value$= 0.00000079 < 0.05$, we reject $H_0 : \theta_s = 1$ in favor of $H_A : \theta_s \neq 1$ at the 5% level.
- There is a strong, positive association between crossing and cars. Elk are more likely to cross if it's a car than a truck zooming overhead.