

**STAT 509 2017 Summer HW10**

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Lecture Day: May 24

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1. Suppose  $X \sim \text{Poisson}(\lambda)$  and we want to find the value of  $\lambda$ . We collect a sample of 20 observations:

$\{4, 2, 1, 3, 5, 3, 0, 2, 3, 1, 4, 2, 2, 2, 2, 0, 4, 4, 1\}$

- (a) Because  $E(X) = \lambda$ , it is reasonable to use **sample mean** as a point estimator to estimate the value of  $\lambda$  (population mean). Use R to get the value of the estimate.
- (b) Because  $X$  is a Poisson random variable, we have  $\text{var}(X) = \lambda$ . Therefore, it is also reasonable to use **sample variance** as a point estimator to estimate the value of  $\lambda$  (population variance). Use R to get the value of the estimate.
- (c) Some day the oracle tells you the true value of  $\lambda$  is 2.5. Which estimate is better in this case, sample mean or sample variance?
2. The electrical component's lifetime follows an exponential distribution with a mean time to failure of 6000 hours.
- (a)  $n$  components are randomly chosen, what is the asymptotic distribution for the **average time to failure** for these  $n$  components? (*Hint: Central Limit Theorem*)
- (b) What is the probability that the average time to failure for 500 randomly chosen components will be less than 5800 hours? (*Hint: Sample size 500 is considered large enough to use CLT*)
3. Shiwen decides to eat some chocolates, which has  $p$  chance to be made by Carolina Reaper.  $p$  is unknown. Shiwen randomly eats 8 chocolates, in which 1 of them makes him suffer.
- (a) Find a point estimate of  $p$ .
- (b) What is the standard error of the point estimator  $\hat{p}$ ?
- (c) Shiwen is not satisfied with the precision level of this result. By the end of the day, he eats 100 chocolates, in which 6 make him suffer. Find a point estimate of  $p$  using 100 observations, and find the standard error. Is the standard error smaller?