STAT 509 2017 Summer HW8
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Lecture Day: May 18

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1. The number of calls received by a telephone answering service follows a Poisson distribution. The calls average 20 per hour.
   (a) What is the probability that 30 calls will arrive in a given 2 hour period?
   (b) What is the probability of waiting more than 15 minutes between two calls? Use both Poisson and Exponential distribution to find the answer. (*Hint: 15 minutes = 0.25 hour*).

2. Suppose X has an exponential distribution with a **expectation** 10. Calculate $P(X < 15|X > 10)$. (*Hint: apply the lack of memory property*)

3. Explosive devices used in mining operations produce (nearly) circular craters when detonated. The radii of these craters, say, Y, follow an exponential distribution with $\lambda = 0.10$.
   (a) Find the proportion of radii that will exceed 20 meters.
   (b) Find the probability that a single denotation will produce a radius between 5 and 15 meters.
   (c) The area of the crater is $W = \pi Y^2$. Find the expected (mean) area produced by the explosive devices; that is, compute $E(W)$.

4. For a type of airplane, the time to maintaince, Y (measured in weeks), varies according to the following pdf:
$$f_Y(y) = ce^{-y/4}, \qquad y > 0$$
   (a) What is the value of c? (*Hint: Is this an exponential distribution?*)
   (b) Calculate $E(Y)$ and $E(Y^2)$.
   (c) Let t be a **fixed** constant. Show that, for $t < \frac{1}{4}$,
$$E(e^{tY}) = \int_0^\infty e^{ty} f_Y(y)dy = \frac{1}{1-4t}.$$
   Note that if you take the derivative of $E(e^{tY})$ with respect to t, and then evaluate this derivative at $t = 0$, you get an answer that matches the value of $E(Y)$ in (b). Verify this statement. This function $E(e^{tY})$ is called the *moment-generating function* of Y. How do you think you could calculate $E(Y^2)$ using the moment-generating function? How about $E(Y^3)$? How about $E(Y^k)$ any an arbitrary postive integer? (You might realize that $E(e^{tY})$ is basically the Laplace transform of the pdf $f_Y(y)$.)

5. An article in *Financial Markets Institutions and Instruments* modeled average annual losses (in billions of dollars) of the Federal Deposit Insurance Corporation (FDIC) with a Weibull distribution with parameters $\delta = 1.9317$ and $\beta = 0.8472$. **Use R** to determine the following:
   (a) Probability of a loss greater than 2 billion.
   (b) Probability of a loss between 2 and 4 billion.
   (c) Mean and variance of loss. (Hint: R command for Gamma function is `gamma()`.)