Design of Experiments

- Factorial experiments require a lot of resources
- Sometimes real-world practical considerations require us to design experiments in specialized ways.
- The **design** of an experiment is the specification of how treatments are assigned to experimental units.

**Goal:** Gain maximum amount of reliable information using minimum amount of resources.

- Reliability of information is measured by the **standard error** of an estimate.
- How to decrease standard errors and thereby increase reliability?

- Recall the One-Way ANOVA:
- Experiments we studied used the Completely Randomized Design (CRD).
• The estimate of $\sigma^2$ was MSW. This measured the variation among responses for units that were treated alike (measured variation within groups).

• We call this estimating the experimental error variation.

• What if we divide the units into subgroups (called blocks) such that units within each subgroup were similar in some way?

• We would expect the variation in response values among units treated alike within each block to be relatively small.

Randomized Block Design (RBD)

• RBD: A design in which experimental units are divided into subgroups called blocks and treatments are randomly assigned to units within each block.

• Blocks should be chosen so that units within a block are similar in some way.

• Reasons for the variation in our data values:

CRD  RBD
• Benefits of a reduction in experimental error:
  • decreases MSW (denominator of F* ratios used in F-tests) → more power to reject null hypotheses
  • decreases standard errors of means → shorter CIs for mean responses

Example 1: Suppose we investigate whether the average math-test scores of students from 8 different majors differ across majors.
  • But … students will be taught by different instructors.
  • We’re not as interested in the instructor effect, but we know it adds another layer of variability.

Solution:

Example 2: Lab animals of a certain species are given different diets to determine the effect of diet on weight gain.
  • Possible block design:
Example 3: An industrial experiment is conducted over several days (with a different lab technician each day).
● Possible block design:

Example 4: (Table 10.2 data)
\( Y = \) wheat crop yield
experimental units = plots of wheat
treatments = 3 different varieties of wheat
blocks = regions of field

Possible arrangement:
• The data are given in Table 10.2.

• Note: Variety A has the greatest mean yield, but there is a sizable variation among blocks.

• If we had used a CRD, this variation would all be experimental error variance (inflates MSW).

• Analysis as CRD (ignoring blocks):

• But … within each block, Variety A clearly has the greatest yield (RBD will account for this).
Formal Linear Model for RBD

• This assumes **one observation per treatment-block combination.**

\[ Y_{ij} = \text{response value for treatment } i \text{ in block } j \]
\[ \mu = \text{an overall mean response} \]
\[ \tau_i = \text{effect of treatment } i \]
\[ \beta_j = \text{effect of block } j \]
\[ \varepsilon_{ij} = \text{random error term} \]

• Looks similar to two-factor factorial model with one observation per cell.

**Key difference:** With RBD, we are not equally interested in both factors.
• The treatment factor is of primary importance; the blocking factor is included merely to reduce experimental error variance.

• With RBD, the block effects are often considered random (not fixed) effects.

• This is true if the blocks used are a random sample from a large population of possible blocks.
• If treatment effects are fixed and block effects are random, the RBD model is called a **mixed model**.

• In this case, the treatment-block interaction is also random.

• This interaction measures the variation among treatment effects across the various blocks.

• The mean square for interaction is used here as an estimate of the **experimental error variance** $\sigma^2$.

**Expected Mean Squares in RBD**

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>E(MS)</th>
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</table>
● Testing for an effect on the mean response among treatments:

\[ H_0: \]

● The correct test statistic is apparent based on \( E(MS) \):

\[ F^* = \quad \text{Reject } H_0 \text{ if:} \]

● Testing for significant variation across blocks:

\[ H_0: \]

● The correct test statistic is again apparent:

\[ F^* = \quad \text{Reject } H_0 \text{ if:} \]

**Example:** (Wheat data – Table 10.2)

● The ANOVA table formulas are the same as for the two-way ANOVA.

● We use software for the ANOVA table computations.
RBD analysis (Wheat data):

\[ F^* = \]

- We conclude that the mean yields are significantly different for the different varieties of wheat. At \( \alpha = 0.05 \), we reject \( H_0: \tau_1 = \tau_2 = \tau_3 = 0 \).

**Note** (for testing about blocks):

\[ F^* = \]

- We would also reject \( H_0: \sigma^2 = 0 \) and conclude there is significant variation among block effects.

- We can again make pre-planned comparisons using contrasts.

**Example:** Is Variety A superior to the other two varieties in terms of mean yield?

\( H_0: \)

\( H_a: \)

Result: