3.8 The Poisson Distribution

- Consider a r.v. \( Y \) that is a count of the number of occurrences of some phenomenon during some fixed unit of time (or space).

Examples: \( Y = \) number of phone calls received per day

\( Y = \) number of spots per square inch of an orange

\( Y = \) number of accidents per week at an intersection

- If we divide this interval of time (or space) into tiny subintervals such that:

and occurrences are independent across subintervals, then we have a _______ experiment.
- The count of occurrences $Y$ is the count of subintervals containing an occurrence.
- Suppose we have $n$ such subintervals.
- Let the mean number of occurrences in the full interval be fixed at and let the number of subintervals $n \to \infty$ (meaning that ).
Then $p(y) =$
Defn. A r.v. $Y$ has a Poisson distribution [Shorthand: $Y \sim \text{Pois}(\lambda)$] if its probability function is:

Theorem: The Poisson distribution is a valid probability distribution.

Proof:

Theorem: If $Y \sim \text{Pois}(\lambda)$, then:

Proof:
The variance result will be proved later.

Example 1: Suppose the number of accidents per month at an industrial plant has a Poisson distribution with mean 2.6. Find the probability that there will be 4 accidents in the next month.

Find the probability of two or more accidents in the next month.
- Tedious cumulative Poisson probabilities can be calculated using Table 3 in Appendix 3 or using R: 

Previous Example: Find the probability of having between 3 and 6 accidents.

What is the probability of 10 accidents in the next half-year?

- Note that

**General Rule**: If the count of the number of occurrences in a unit of time follows a Poisson (θ) distribution, then the count of the number of occurrences in t units of time follows a
Relationship with Binomial Distribution

- If \( n \) is large, \( p \) is small, and \( \lambda = np \) is somewhat small (book: ), then the \( \text{Bin}(n,p) \) probabilities are approximately equal to the \( \text{Pois}(\lambda) \) probabilities.

Example: Suppose 3% of all USC students are vegetarians. Select 100 USC students at random. Find the probability of selecting at least five vegetarians.

Binomial:

Poisson: