Time Series Analysis Project Paper
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Time Series Analysis of Late-Life Immigrants Entering the United States

Abstract

The aims of the present study are to identify a model best fitting the late-life immigrant data and to forecast the number of late-life immigrants entering the United States. The method of maximum likelihood was used to estimate the parameters and to forecast the number of late-life immigrants in the future. The late-life immigrant data from 1933 to 2007, which have been documented annually by the Department of Homeland Security, reveal that the ARI (1, 1) model may fit most adequately and forecast that the number of late-life immigrants who enter the U.S. will reach approximately 540,000 in 10 years, which explains 1.6 times increase, compared to the number of late-life immigrants entering the U.S. in 2007, under the assumption that current political, social, and economical conditions will be persistent in the future. Policy decision makers may be advised to prepare measures and/or programs for the increasing number of late-life immigrants so that they can be smoothly adjusted to their life in the U.S. Under the current welfare reform act which bars those without citizenship from accessing federally-funded public services including Medicaid, the well-being and health of indigent elderly immigrants without citizenship may be at risk when they become ill.
Introduction

Brief History of Immigration Policies in the U.S.

The U.S. Census Bureau has projected that the U.S. population will increase to 392 million by 2050, and that 86 percent of this increase may be due to the effects of net immigration (U.S. Census Bureau, 2008). The number of elderly immigrants 65 years or older has almost doubled since 1990 from 2.7 million to 4.3 million people (Leach, 2008/9). The increasing numbers of older immigrants will be mainly from Latin America and Asia (He, 2002).

Hagan (2004) argued that the impact of immigration is not just demographic but it is economic. Immigrants have played an increasingly important role in the U.S. labor force historically and depended on immigration for its labor-force growth. Foreign-born workers were especially important to the nation’s economic growth and job creation in the mid to late 1990s, accounting for almost half of the net increase in the nation’s civilian labor force from 1996 to 2000 (Hagan, 2004; Mosisa, 2002; Sum, Fogg, Harrington, Khatriwada, Trub’sky, & Palma, 2002).

Even though U.S. immigration policies seemed to have set the parameters for legal immigration, the prominent periods of immigration reform have been inconsistent, at times curtailing and at other times expanding the conditions depending on U.S. economy (Hagan, 2004). For example, the early immigration laws such as the Chinese Exclusion Act of 1882 and the Gentlemen’s Agreement of 1907 reflected not only racial preference but also the economic factor in immigration law because the U.S. was concerned about the influence of inexpensive immigrant labor from Asian countries (Smith & Edmonston, 1997). The immigration policy of the U.S. such as the 1921 Quota Act and the follow-up 1924 also barred Asian immigration, and deported Mexican peoples because there were also fears about adverse consequences for native workers due to their providing cheap labor. During World War II, in an effort to fill labor shortages in the farm industry, the Bracero Program was enacted in 1943, which allowed farm workers from Mexico to work on U.S. farms on a
temporary basis. However, two decades later, this program ended in 1964 due to the critics that Bracero workers were no longer beneficial to the U.S. economy, but rather they were adversely affecting the wages and jobs of native farm workers (Hagan, 2004; Martine, 1996). The establishment of U.S. immigration policy since 1965 had additional purpose, the “social goal of family unity”. By the mid-1970s, large numbers of legal immigrants entered into the U.S. from Latin America along with smaller flows from Asia. The passage of the Immigration and Nationality Act Amendments of 1965, the Hart-Celler Act (PL 89-236) extended the family reunification provision to include parents of citizens (Brunner & Colarelli, 2010).

Family Reunification Program

After the Immigration and Nationality Act Amendments of 1965, on average, late-life immigrants have entered the United States through the extended family reunification program, i.e., at the invitation of their adult children who are naturalized in the U.S. (Brunner & Colarelli, 2010; Gelfand & Yee 1991; Leach, 2008/9). The primary objective of the family reunification program was to bring the nation’s immigration law in line with civil right legislation so that immigrants would not be discriminated against on the basis of ethnicity or nationality.

Personal Responsibility and Work Opportunity Reconciliation Act of 1996

When legal late-life immigrants reached age 65 and older, the late-life immigrants had had the same rights as their U.S.-born counterparts in access to the public assistance system until the implementation of U.S. Public Law 104-193 in 1996, also called the Personal Responsibility and Work Opportunity Reconciliation Act of 1996 (PRWORA) (Nam 2008, 2011; Zimmermann & Tumlin, 1999). However, the PRWORA distinguished between citizens and non-citizens in access to public assistance and, by extension, to healthcare services. As a result, elderly immigrants without citizenship have been barred from accessing Medicaid, except for those living in states that allow ineligible noncitizens to access the state-funded Medicaid program (Fremstad & Cox, 2004; Nam,
By taking this approach, its creators expect PRWORA to conserve more public financing and address deficits in welfare funding by emphasizing the principle of self-sufficiency and by defining the situation in which legal immigrants without citizenship receive public benefits as a social problem (Agrawal, 2008; Nam, 2011).

Profiles of Late-Life Immigrants

The demographic profiles of late-life immigrants imply that they have little or no work history in the United States, and they are more likely than their native counterparts in the U.S. to live in poverty (Leach, 2008/9; Nam, 2008; He, 2002). In addition, the U.S. Census Bureau (He, 2002) revealed that in 1999, older noncitizens had the lowest overall health insurance coverage rate (84.2% for noncitizens as opposed to 92.2% for native born and 98.1% for naturalized citizens). Differences were even more striking in terms of private health insurance coverage (21.4% for noncitizens as opposed to 63.8% for native born and 46.7% for naturalized citizens). However, the Medicaid coverage status of noncitizens was highest (34.5% for noncitizens while only 7.7% for native born and 15.2% for naturalized citizens).

Potentially Negative Impact of the PRWORA on Late Life Immigrants

The statistics described above suggest that elderly immigrants who were covered by health insurance were more likely to rely on Medicaid due to their socioeconomic status in the United States. Consequently, the PRWORA has caused great concern about the health of elderly immigrants and their well-being as they fall into poverty. This concern is heightened by the fact that most immigrants who arrived after August 22, 1996 when the PRWORA was implemented are no longer eligible for the major source of health insurance on which these immigrants appear to rely, Medicaid, until they become citizens. The only exception would be if individual states are willing to pay for the health care services of elderly immigrants living their respective jurisdictions (Fremstad & Cox, 2004; Nam, 2011; Smith, 2001).
To become U.S. citizens, immigrants need to pass the naturalization process in English or seek employment to meet the 40-quarter work requirement (10 years of work history in the U.S.) to be eligible for public assistance (Nam, 2011; National Immigration Law Center, 2002). For many elderly immigrants, these are unrealistic and unmanageable demands. In addition, some racial/ethnic minority groups may become acculturated to life in the U.S. more easily by using the English language and/or participating in the mainstream culture of the U.S.; however, others may not be as successful in acculturating to the host country. In addition, in relation of elderly immigrants’ health to the PRWORA, Binstock and Jean-Baptiste (1999) observed in a qualitative study that the stresses associated with the loss of public benefits had already affected the mental health of elderly immigrants in Dade County, Florida. It is assumed that without measures to curb the number of late-life immigrants entering the U.S. or extended public service programs for the late-life immigrants to acquire the U.S. citizenship, the health of the late-life immigrants may be at risk.

The aims of the present study are 1) to identify a possible candidate model for the late-life immigrant data and 2) to forecast the number of late-life immigrants entering the U.S. under the assumptions that the current immigration policy and welfare reform are persistent in the future. Forecasting the number of late-life immigrants entering the U.S. annually will help policy decision makers prepare the U.S. immigration and welfare policies for the influx of the late-life immigrants. The term of “late-life immigrants” in the present study refers to any legal permanent immigrants who migrated to the U.S. at age 45 or older.

Data

The present study used immigrant data released by the Department of Homeland Security (DHS). Even though the U.S. government has published data on immigration annually since the 1890s (DHS, 2010), the data before 1996 are not available directly from their homepage. The author of the
present study made contact to the Office of Immigrant Statistics at DHS via email and obtained the data from 1933 to 1995. The DHS distinguishes between legal permanent residents and refugees/asylees (DHS, 2010). Legal permanent residents refer to those who have been granted lawful permanent residence in the United States (i.e., “green card” recipients) (DHS, 2010).

The late-life immigrant data in this present study consist of 75 observations, that is to say, one observation from each year from 1933 to 2007. The last three observations, i.e., data from 2008 to 2010, were left for the further diagnosis of the accuracy of a model which will be specified through the process of this study. Following sections include 1) model specification, 2) model fitting and diagnostics, 3) forecasting, and 4) Discussion. All statistical analyses will be performed in R.

Model Specification

The plot in Figure 1 on the page 7, which is a scatterplot of $Y_t = \text{value of the variable Y (Annual number of late-life immigrants entering the U.S.) at time } t \text{ (Years)}$, shows the number of late-life immigrants has been increasing, behaving a quadratic pattern with median = 49,729 (ranging from 3,904 to 370,652). More specifically, there was a slow increase in the number of late-life immigrants from 1933 to 1980; there has been a steep increase afterwards with a big spike in early 1990s. Even though there were some levels of drops after 1996, the increasing trend continued after 2000. Overall, the late-life immigrant data may be represented as a realization of a deterministic trend with a quadratic function. The deterministic trend can be removed by using a “detrending” approach. However, this approach requires an assumption that the trend lasts “forever” and ignores the correlation among time lags since the deterministic trend models are based on the least squares model fit (Cryer & Chan, 2008). Hence, the present study will select a “differencing” approach, which is also called ARIMA (p, d, q) models, for the late-life immigrant data, which is developed...
extensively by Box and Jenkins (Cryer & Chan, 2008). In ARIMA \((p, d, q)\), the character “\(p\)” refers to the order of the autoregressive component, “\(d\)” the number of differences needed to arrive at a stationary ARMA \((p, q)\) process, and “\(q\)” the order of the moving average component. This approach is to apply differencing repeatedly to the series \(\{Y_t\}\) until the differenced observations resemble a realization of a stationary time series. Securing stationarity by using the differencing approach will allow for the use of theory of stationary processes for the modeling, analysis, and forecasting of the stationary series. The general form of ARIMA \((p, d, q)\) models in backshift notations is

\[
\Phi(1 - B)^d Y_t = \Theta B \epsilon_t
\]
where \( \{e_t\} \) is zero mean white noise with variance \( \text{var} (e_t) = \sigma_e^2 \). In this notation, a stochastic process \( \{Y_t\} \) is believed to follow an autoregressive integrated moving average model with \( d \)th differences. The letter “B” refers to the backshift operator. For example, \((1-B)Y_t = Y_t - Y_{t-1}\).

The autoregressive (AR) and moving average (MA) characteristic operators are:

\[
\varphi(B) = (1 - \varphi_1 B - \varphi_2 B^2 - \cdots - \varphi_p B^p), \text{ i.e., Autoregressive (AR) characteristic operator}
\]

\[
\theta(B) = (1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_p B^p), \text{ i.e., Moving average (MA) characteristic operator.}
\]

**Box-Cox Power Transformation**

Before taking a difference for stationary process, a possible non-constant variance needs to be stabilized through a power transformation introduced by Box and Cox (1964). The power transformation will also frequently improve an approximation of normality. The transformation is defined by
\[
T(Y_t) = \begin{cases} 
\frac{Y_t^\lambda - 1}{\lambda}, & \lambda \neq 0 \\
\ln(Y_t), & \lambda = 0,
\end{cases}
\]

where \(\lambda\) is called the transformation parameter.

Guided by Box and Cox (1964), a \(\lambda\) value is identified as in Figure 2 on the page 8. An approximate 95% confidence interval for \(\lambda\) includes \(\lambda = 0\). As in the formula above, \(\lambda = 0\) requires the logarithm transformation, \(T(Y_t) = \ln(Y_t)\), by L'Hoptial's Rule (Cryer & Chan, 2008), which is constructed using the large sample properties of the maximum likelihood estimates. That is, the value of \(\lambda = 0\) maximizes the normal log-likelihood function in the data used for the present study.

Taking a Difference

The plot of the transformed data with logarithm in Figure 3 on the page 9 shows an approximately increasing trend with lots of momentum between observations, which suggests non-stationarity in the data. A quantitative test, the augmented Dickey-Fuller unit-root test, for stationarity guided by Dickey and Fuller (1979) also supports that the observed time series is not stationary (\(P - value = 0.1\)) in testing \(H_0: \alpha = 1\) (nonstationary) (See Output-1 in the appendix). The observed increasing trend and the augmented Dickey-Fuller unit-root test result recommend taking a difference of the log-transformed data for stationarity. Output-2 in the appendix reports that the log-transformed late-life immigrant data came to be stationary after the first differencing (\(P - value = 0.01\)). This result indicates that we have sufficient evidence that this series is stationary at the \(\alpha = 0.05\) level. Theoretically to say, the AR characteristic polynomial \(\varnothing^*(B) = \varnothing(B)(1 - \alpha B)\) does not contain a unit root, so \(\{Y_t - Y_{t-1}\}\) or \(\{\nabla Y_t\}\) is stationary. The scatter plot after taking the first difference of the transformed data in Figure-4 also support the series is now stationary.
Figure 4: Scatter Plot after Taking a First Difference of the Transformed Data

Model Specifications

For selecting an ARMA (p, q) model for the differenced data, the sample ACF and PACF were plotted. According to Figure 5 in the appendix, the plots show the fluctuations of the sample autocorrelation values $r_k$ within $\pm \frac{2}{\sqrt{T}} = 0.23$ which is margin of random error bounds, suggesting a white noise process. The sample EACF in Output-3 in the appendix and BIC plots in Figure 6 in the appendix, however, show conflict results: the sample EACF supports an AR (1) model since a ‘wedge’ with a tip is at (1, 0) for the first differenced data, while the BIC output supports an AR (2) model appropriately fits the differenced data. Based on the results, candidate models for the late-life immigrant data may be ARI (1, 1), ARI(2, 1), or ARIMA (0, 1, 0) model. Hence, the present study will start to fit and diagnose ARI (1, 1) model, followed by possible candidates including ARI (2, 1), and ARIMA (0, 1, 0) models to select the best model for the late-life immigrant data. Even though
an IMA (1, 1) model was not suggested by the assessments for a model specification, the IMA (1, 1) model will also be tested to comprehensively investigate all possible candidates.

**Model Fitting and Diagnostics**

**Model Fitting**

An ARIMA (p, d, q) process with p = 1, d = 1, and q = 0 is called an ARI (1, 1) process and can be expressed as

\[(1 - \phi B)(1 - B)Y_t = e_t\]

Or, equivalently,

\[Y_t - Y_{t-1} = \phi (Y_{t-1} - Y_{t-2}) + e_t,\]

where \(\{e_t\}\) is a normal zero mean white noise process with \(\text{var}(e_t) = \sigma_e^2\). Since the late-life immigrant data were transformed by logarithm, the formula will be changed as follows:

\[\log Y_t - \log Y_{t-1} = \phi (\log Y_{t-1} - \log Y_{t-2}) + e_t.\]

To estimate unknown parameters, the method of maximum likelihood (ML) was used since the ML method in fitting time series has some advantages including 1) that parameter estimates are based on the entire observed sample \(Y_1, Y_2, \ldots, Y_n\); and 2) ML estimators have very nice large-sample distributional properties (Cryer, & Chan, 2008). As in Output-4 in the appendix, the estimated \(\hat{\phi} = 0.1999\) and estimated standard error \(\text{SE}(\hat{\phi}) = 0.1138\), where \(\hat{\phi} \sim AN \left(\phi, \frac{1-\phi^2}{n}\right)\), gives estimated parameters as follows;

\[\log Y_t - \log Y_{t-1} = 0.1999(\log Y_{t-1} - \log Y_{t-2}) + e_t.\]

Using the ML method, the approximate large-sample confidence interval for ARI (1, 1) model based on the parameter estimates is

\[\hat{\phi} \pm Z_{\frac{1}{2}} \cdot \text{SE}(\hat{\phi})\]
\[ = 0.1999 \pm 1.96(0.1138) \Leftrightarrow (-0.023148, 0.422948) \]

The interval includes zero, indicating that \( \hat{\theta} \) is not statistically different from zero. Based on this result, no more complex AR-type model, i.e., ARI (2, 1), needs to be examined.

**Model Diagnostics**

To check the fit of the ARI (1, 1) model, the observed residuals \( \hat{\varepsilon}_t \) will be examined since the residuals serve as proxies for the white noise terms \( e_t \). If the model is correctly specified and the estimates are reasonably close to the true parameters, the residuals should behave roughly like a sequence of independent, normal random variables with zero mean and constant variance.

Commonly, the residuals will be standardized, that is, \( \hat{\varepsilon}_t^* = \frac{\hat{\varepsilon}_t}{\hat{\sigma}_e} \), where \( \hat{\sigma}_e^2 \) is an estimate of the white noise error variance \( \sigma_e^2 \). Most of the standardized residuals \( \{\hat{\varepsilon}_t^*\} \) are assumed to fall between -3 and 3.

In checking whether the residuals behave like a white noise process and the ARI (1, 1) model fit the data adequately, a series of tests will include checking for the normality and independence assumptions and the Ljung-Box test.

**Normality Assumption** – In order to visually assess the normality assumption, the histograms and QQ plots of the standardized residuals are used. For the hypothesis tests for normality, the Shapiro-Wilk test is used. As in Figure 7 in the appendix, the histogram and QQ plots show no gross departures from normality. The Shapiro-Wilk test in Output-5 in the appendix supports the results of the plots: In testing \( H_0: \) "the standardized residuals are normally distributed," with the observed P-value of 0.09157, we failed to reject the null hypothesis at \( \alpha = 0.05 \) level. In summary, there is not sufficient evidence against normality.

**Independence Assumption** – For visual assessment of the independent assumption, a time series plot of the standardized residuals is used, and a runs test with the standardized residuals is used for a formal test of the independence assumption. The standardized residual plot displays no discernible
patterns and looks to be random in appearance. The runs test supports the visual assessment: In testing $H_0$: “the standardized residuals are independent,” with the observed P-value of 0.605, we failed to reject the null hypothesis at $\alpha = 0.05$ level. In summary, there is no evidence against independence.

**Ljung-Box Test** - To further check the adequacy of a fitted ARI (1, 1) model formally, the Ljung-Box test (1978) is used. According to Figure 9 in the appendix showing the results of the modified Ljung-Box test, in testing $H_0$: “the ARI (1, 1) model is appropriate,” we do not have sufficient evidence against ARI (1, 1) model adequacy for the log-transformed late-life immigrant data when the maximum lag $K = 10$ since we observed $\chi^2_{90,0.05} = 10.0538$ and its corresponding $p$-value $= 0.3461$. The graphically displayed Ljung-Box test also supports the aforementioned hypothesis test result: 1) the residuals plotted through time displayed on the top of Figure 9 also show that the standardized residuals fall between -3 and 3, suggesting the residuals are normally distributed; 2) the sample autocorrelation function (ACF) of the residuals in the middle of the same figure shows that the residuals are approximately uncorrelated, behaving a white noise process; and 3) the p-values of the modified Ljung-Box test for various values of $K$ on the bottom displays that all of the modified Ljung-Box test p-values are larger than $\alpha = 0.05$.

**Overfitting** - The initial model specification processes presented conflict models. Hence, to further assess the validity of the ARI (1, 1) model, the ARI (2, 1), IMA (1, 1), and ARIMA (0, 1, 0) models are assessed by examining the significance of the additional parameter estimates and the change in the estimates from the assumed model and by assessing the diagnostic plots.

Output-7 in the appendix presents the IMA (1, 1) model fit and diagnostics with standardized residuals. We see that a 95% confidence interval for $0_1$, the IMA model parameter, is

$-0.1917 \pm 1.96(0.1054) \iff (-0.398284, 0.014884)$,
Table 1: Comparisons of Diagnostics among the Suggested Models

<table>
<thead>
<tr>
<th></th>
<th>ARI (1, 1)</th>
<th>IMA (1, 1)</th>
<th>ARIMA (0, 1, 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence Intervals</td>
<td>Insignificant</td>
<td>Insignificant</td>
<td>N/A</td>
</tr>
<tr>
<td>AIC</td>
<td>-90.94</td>
<td>-90.9</td>
<td>-89.93</td>
</tr>
<tr>
<td>Whitenoise Variance</td>
<td>0.06544</td>
<td>0.06549</td>
<td>0.0682</td>
</tr>
<tr>
<td>Diagnostics</td>
<td>Normality&lt;sup&gt;1&lt;/sup&gt; ( p = 0.092 )</td>
<td>( p = 0.089 )</td>
<td>( p = 0.078 )</td>
</tr>
<tr>
<td></td>
<td>Independence&lt;sup&gt;2&lt;/sup&gt; ( p = 0.605 )</td>
<td>( p = 0.890 )</td>
<td>( p = 0.211 )</td>
</tr>
<tr>
<td></td>
<td>Ljung-Box Test&lt;sup&gt;3&lt;/sup&gt;</td>
<td>Fail to reject</td>
<td>Fail to reject</td>
</tr>
</tbody>
</table>

Note:
The Shapiro-Wilks test was used for the formal normality assumption test.
The runs test was used for the formal independence assumption test.
The modified Ljung-Box Test is testing \( H_0: \) “the suggested model is appropriate.”

which also include zero. Therefore, \( \hat{\theta}_1 \) is not statistically different than zero, which also suggests that the IMA (1, 1) is not necessary. The standardized residuals are not against normality and independence assumptions (See Figure 10 and Output-7 in the appendix) and the Ljung-Box test tells IMA (1, 1) model adequately fits the data (see Figure 11 in the appendix).

Output-8 in the appendix shows the ARIMA (0, 1, 0) model diagnostics with residuals. After taking a first difference, the residual plots in Figure 12 and Output-8 in the appendix do not show serious violation of normality and independence assumptions. However, the Ljung-Box test results do not support that the ARIMA (0, 1, 0) model adequately fits the late-life immigrant data: many of the p-values of the modified Ljung-Box test for various values of \( K \) on the bottom in Figure 13 in the appendix are less than \( \alpha = 0.05 \).

Table 1 presents the summary of the diagnostic values of the suggested models. Even though the ARIMA (0, 1, 0) model has the smallest AIC with a trivial difference, the estimate of the white noise variance is greatest among the three models. In addition, the Ljung-Box test strongly discounts the ARIMA (0, 1, 0) model. The IMA (1, 1) model turned out to be as a strong candidate model as the ARI (1, 1) model since all results from the diagnosis of the two models are very similar.
Comparing the forecasts displayed in Output-9, 10, and 11 in the appendix, the predicted values from the MMSE forecast of $Y_{2008}$, $Y_{2009}$, and $Y_{2010}$ in the ARI (1, 1), IMA (1, 1) and ARIMA (0, 1, 0) models are also very similar each other: the values from the three models are ranging from approximately 340,000 in 2007 to approximately 370,000 in 2010. In addition, all of them are showing an increasing trend over times.

Taking into account all the results from the diagnostics in the table as well as the sample EACF and BIC plot which were used for the initial model specification, the ARI (1, 1) model may be a better model than the IMA (1, 1) or the ARIMA (0, 1, 0) model since the plots favored the AR-type models. The number of late-life immigrants in the future will be forecasted through the ARI (1, 1) model in the next section.

**Forecasting**

To compare the forecasts through the suggested models to the actual values of the process, the last three observations, which are the values from the years of 2008, 2009, and 2010, are withheld. This technique will provide further information on how accurate the forecasting is. For forecasting, the minimum mean squared error (MMSE) forecast is adopted as a formal mathematical criterion to calculate model forecasts. The criterion is based on the mean squared error of prediction, i.e.,

$$\text{MSEP} = \mathbb{E}\{[Y_{t+\ell} - h(Y_1, Y_2, \ldots, Y_t)]^2\}.$$  

To minimize the MSEP,

$$h(Y_1, Y_2, \ldots, Y_t) = \mathbb{E}(Y_{t+\ell}|Y_1, Y_2, \ldots, Y_t).$$

Hence, the $\ell$-step ahead forecast is

$$\hat{Y}_t(\ell) = \mathbb{E}(Y_{t+\ell}|Y_1, Y_2, \ldots, Y_t),$$

which is the MMSE forecast of $Y_{t+\ell}$.

As in Output-4, the fitted ARI (1, 1) models was
The estimated forecasts and standard errors both on the log scale and on the original scale are given for lead times $\ell = 1, 2, \ldots, 10$ as in Output-9 in the appendix. The $\ell$-step ahead MMSE forecast of $Y_{t+\ell}$ on the log scale is back-transformed to the original scale by using the following formula:

$$Y_t(\ell) = \exp \left( Z_t(\ell) + \frac{1}{2} \text{var}[e_t(\ell)] \right),$$

where $\text{var}[e_t(\ell)]$ is the variance of the $\ell$-step ahead forecast error $e_t(\ell) = Z_{t+\ell} - Z_t$, and $Z_t = \ln Y_t$. However, a 100(1 - $\alpha$) percent prediction interval for $Y_{t+\ell}$ is formed by exponentiating the endpoints of the prediction interval for $Z_{t+\ell} = \log Y_{t+\ell}$. In summary, a 100(1 - $\alpha$) percent prediction interval for $Y_{t+\ell}$ is

$$1 - \alpha = \text{pr}(\hat{Z}_{t+\ell}^{(L)} < Z_{t+\ell} < \hat{Z}_{t+\ell}^{(U)}) = \text{pr}(e^{\hat{Z}_{t+\ell}^{(L)}} < Y_{t+\ell} < e^{\hat{Z}_{t+\ell}^{(U)}}).$$

The predicted values from the MMSE forecast of $Y_{t+\ell}$ for the late-life immigrant data and the actual values of the late-life immigrant data are compared.

As Table 2 above presents, the predicted values from the MMSE forecast of $Y_{2008}$, $Y_{2009}$, and $Y_{2010}$ are very similar to the actually observed values of the late-life immigrant data, which suggests the ARI (1, 1) model explains adequately the late-life immigrant data. According to the predicted values from the MMSE forecast, in 10 years from 2007, the number of late-life immigrants who

<table>
<thead>
<tr>
<th>Year</th>
<th>Observed</th>
<th>$e_{t+\ell}^{(L)}$</th>
<th>$\hat{Y}_t(\ell)$</th>
<th>$e_{t+\ell}^{(U)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>356316</td>
<td>200420</td>
<td>341907</td>
<td>546327</td>
</tr>
<tr>
<td>2009</td>
<td>353573</td>
<td>141229</td>
<td>358462</td>
<td>724278</td>
</tr>
<tr>
<td>2010</td>
<td>329400</td>
<td>121763</td>
<td>376970</td>
<td>899612</td>
</tr>
</tbody>
</table>

$\log Y_t - \log Y_{t-1} = 0.1999(Y_{t-1} - Y_{t-2}) + e_t,$

so that $\hat{\theta} = 0.1999$ and the white noise variance estimate $\hat{\sigma}^2 = 0.06544$. The predicted values from the MMSE forecast of $Y_{t+\ell}$ for the late-life immigrant data and the actual values of the late-life immigrant data are compared.
enter the U.S. will reach approximately 540,000, which accounts for 1.6 times increase. Without taking measures for the late-life immigrants, some racial/ethnic minority groups may not be successfully acculturated to the life in the U.S. and may not successfully achieve citizenship status to be eligible for the public medical services when they fall into poverty. Specifically, without taking actions for the late-life immigrants who may be disadvantaged by the PRWORA, widened discrepancies in the quality of life and health among racial/ethnic groups in the U.S. can be easily anticipated.

**Discussion**

By using the method of maximum likelihood, the present study identified that the ARI (1, 1) model fits the late-life immigrant data most adequately. The estimated parameters are
\[ \log Y_t - \log Y_{t-1} = 0.1999(\log Y_{t-1} - \log Y_{t-2}) + e_t. \]

According to the predicted values from the MMSE forecast, in the near future, in 2017, the number of late-life immigrants who enter the U.S. will reach approximately 540,000, which accounts for 1.6 times increase compared with the number of late-life immigrants who enter the U.S. in 2007. The present study used national data documented by the DHS, but the selected model and forecasting can be generalizable only under some assumptions including that the current immigration policy, welfare reform, and supply & demand for foreign labor forces in the U.S. are persistent in the future.

Another limitation is that the approximate large-sample confidence interval of \( \hat{\theta} \) for ARI (1, 1) includes zero, indicating that \( \hat{\theta} \) is not statistically different from zero. However, no other models including IMA (1, 1), and ARIMA (0, 1, 0) turned out to be better than ARI (1, 1).

Defining the term “late-life immigrants” as those who enter the U.S. at age 45 or older may also be a limitation. Compared to early twentieth century, the age of 45 or older is no longer “old” in present time since they actively participate in social and economic activities even after immigration to the U.S., which implies they have more chance to achieve citizenship status than those aged 55 or older. Hence, immigrants aged 55 or older may be a better target population for the present study. However, the last age category in the DHS data had been “45 or older” until the DHS documented for the 1940 DHS data.
References


Appendix

Figure-1: Late-Life Immigrant Original Data

Figure-2: Box-Cox plot

Figure-3: Plot of Log-Transformed Data

Figure-4: First Differencing of the Log-Transformed Data
Figure 5: Sample ACF and PACF

![Sample ACF for the Data after First-Difference](image1)
![Sample PACF for the Data after First-Difference](image2)

Figure 6: BIC

![BIC Diagram](image3)
Figure 7: Normality Assumption Test with Residuals - ARI (1, 1) Model

Figure 8: Plot for Independence Assumption Checking with Residuals - ARI (1, 1) Model
Figure-9: Ljung-Box Test - ARI (1, 1) Model
Figure-10: Normality & Independence Assumption Checking - IMA (1, 1) Model

Figure-11: Ljung-Box Test - IMA (1, 1) Model
Figure-12: Normality & Independence Assumption Checking - ARIMA (0, 1, 0) Model

Figure-13: Ljung-Box Test - ARIMA (0, 1, 0) Model
Figure-14: Prediction Interval Plot on Log Scale
Output-1: Augmented Dickey-Fuller Unit Root test with the Log-Transformed Data

> ar(diff(immig45.log))

Call:
ar(x = diff(immig45.log))

Order selected 0  sigma^2 estimated as  0.06549

> ADF.test(immig45.log,selectlags=as.list(0),itsd=c(1,0,0))

--------- ------ ------- -----
Augmented Dickey & Fuller test
--------- ------ ------- -----

Null hypothesis: Unit root.
Alternative hypothesis: Stationarity.

----

ADF statistic:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| adf.reg  | -0.04      | 0.024   | -1.646   | 0.1      |

Lag orders: 0
Number of available observations: 74
Warning message:
In interpolpval(code = code, stat = adfreg[, 3], N = N):
p-value is greater than printed p-value

➢ In testing
H₀: α = Nonstationary
vs.
H₁: α = Stationary,
with p-value = 0.1 for the augmented Dickey-Fuller unit-root test for difference nonstationarity, we do not have sufficient evidence that this series is stationary at the α = 0.05 level.
Theoretically to say, the AR characteristic polynomial \( \Phi^*(B) = \Phi(B)(1 - \alpha B) \) contains a unit root. In other words, \( \{Y_t\} \) is nonstationary, but \( \{\nabla Y_t\} \) is stationary.
Output-2: Augmented Dickey-Fuller unit root test after taking a first difference of the log-transformed data

> ar(diff(immig45.log.diff))

Call:
  ar(x = diff(immig45.log.diff))

Coefficients:
  1     2     3     4     5     6     7     8
-0.6876 -0.5868 -0.6623 -0.5432 -0.4557 -0.4417 -0.3870 -0.2372

Order selected 8  sigma^2 estimated as  0.07795
> ADF.test(immig45.log.diff,selectlags=as.list(1:8),itsd=c(1,0,0))

--------- ------ ------ ------ -----
Augmented Dickey & Fuller test
--------- ------ ------ ------ -----

Null hypothesis: Unit root.
Alternative hypothesis: Stationarity.

-----

ADF statistic:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| adf.reg  | -1.11      | 0.188   | -5.892   | 0.01     |

Lag orders: 1 2
Number of available observations: 71
Warning message:
In interpolpval(code = code, stat = adfreg[, 3], N = N) :
p-value is smaller than printed p-value

➢ In testing
  H₀: α = Nonstationary
  vs.
  H₁: α = Stationary,
  with p-value = 0.01 for the augmented Dickey-Fuller unit-root test for difference nonstationarity, we have sufficient evidence that this series after taking a first difference of the log-transformed data is stationary at the α = 0.05 level.
Output-3: Sample EACF

\[
\begin{align*}
\text{> eacf(immig45.log)} \\
\text{AR/MA} \\
0 & 1 2 3 4 5 6 7 8 9 10 11 12 13 \\
0 & xxxxxxxxxxxxx x x x \\
1 & o o o o o o o o o o o o \\
2 & o o o o o o o o o o o o \\
3 & o x o o o o o o o o o o \\
4 & o x x o o o o o o o o o o \\
5 & o o x o o o o o o o o o o \\
6 & x o o o o o o o o o o o o \\
7 & x x o o o o o o o o o o o
\end{align*}
\]

Output-4: ARI (1, 1) Model Fitting

\[
\begin{align*}
\text{> immig45.log.fit=arima(immig45.log,order=c(1,1,0),method='ML')} \\
\text{> immig45.log.fit} \\
\text{Call:} \\
\text{arima(x = immig45.log, order = c(1, 1, 0), method = "ML")} \\
\text{Coefficients:} \\
\text{ar1} \\
0.1999 \\
\text{s.e.} 0.1138 \\
\text{sigma^2 estimated as 0.06544: log likelihood = 46.47, aic = -90.94} \\
\end{align*}
\]

\[
\begin{align*}
\text{Significance test for ARI (1,1):} \\
0.1999 \pm 1.96 \times 0.1138 = (-0.023148, 0.422948) \\
The \text{AR parameter estimate} \hat{\theta} \text{ is not significantly different from zero.}
\end{align*}
\]
Output-5: Shapiro-Wilk Test for Normality – ARI (1, 1) Model

> shapiro.test(rstandard(immig45.log.fit))

    Shapiro-Wilk normality test

data:  rstandard(immig45.log.fit)
W = 0.9717, p-value = 0.09157

Output-6: Runs Test for Independence – ARI (1, 1) Model

> runs(rstandard(immig45.log.fit))

$pvalue
[1] 0.605

$observed.runs
[1] 33

$expected.runs
[1] 35.56

$n1
[1] 27

$n2
[1] 48

$k
[1] 0

>
Output-7: Overfitting (2): IMA (1, 1) Model and Checking Assumptions

```r
> immig45.log.ima_1 <- arima(immig45.log, order = c(0, 1, 1), method = "ML")
> immig45.log.ima_1

Call:
  arima(x = immig45.log, order = c(0, 1, 1), method = "ML")

Coefficients:
  ma1
      0.1917
s.e.  0.1054

sigma^2 estimated as 0.06549:  log likelihood = 46.45,  aic = -90.9

> shapiro.test(rstandard(immig45.log.ima_1))

  Shapiro-Wilk normality test

data:  rstandard(immig45.log.ima_1)
  W = 0.9716, p-value = 0.08968

> runs(rstandard(immig45.log.ima_1))

$pvalue
[1] 0.89

$observed.runs
[1] 35

$expected.runs
[1] 34.97333

$n1
[1] 26

$n2
[1] 49

$k
[1] 0

- Significance test for IMA (1,1):
  -0.1917 ± 1.96 * 0.1054 = (-0.398284, 0.014884)

  The MA parameter estimate \( \hat{\theta} \) is not significantly different from zero.
```
Output-8: Overfitting (3): ARIMA (0, 1, 0) Model and Checking Assumptions

> immig45.log.diff=arima(immig45.log,order=c(0,1,0),method='ML')
> immig45.log.diff

Call:
arima(x = immig45.log, order = c(0, 1, 0), method = "ML")

sigma^2 estimated as 0.0682: log likelihood = 44.96, aic = -89.93
>
> shapiro.test(rstandard(immig45.log.diff))

Shapiro-Wilk normality test
data: rstandard(immig45.log.diff)
W = 0.9706, p-value = 0.07764

> runs(rstandard(immig45.log.diff))
$pvalue
[1] 0.211

$observed.runs
[1] 29

$expected.runs
[1] 34.33333

$n1
[1] 25

$n2
[1] 50

$k
[1] 0

>
Output-9: Forecasting ARI (1, 1) Model

> #Prediction
> immig45.log.predict=predict(immig45.log.fit,n.ahead=10)
> round(immig45.log.predict$pred,3)
Time Series:
Start = 2008
End = 2017
Frequency = 1
> round(immig45.log.predict$se,3)
Time Series:
Start = 2008
End = 2017
Frequency = 1
[1] 0.256 0.400 0.510 0.602 0.681 0.753 0.818 0.878 0.935 0.988
> log_back.predict=round(exp(immig45.log.predict$pred+(1/2)*(immig45.log.predict$se)^2),3)
> log_back.predict
Time Series:
Start = 2008
End = 2017
Frequency = 1
[1] 341907.9 358462.9 376970.0 396680.0 417472.7 439366.3 462410.4 486663.7
[9] 512189.1 539053.3
> #Prediction Interval
> immig45.UPI=(immig45.log.predict$pred)+(qnorm(0.975,0,1)*immig45.log.predict$se)
> immig45.LPI=(immig45.log.predict$pred)-(qnorm(0.975,0,1)*immig45.log.predict$se)
> Year=c(2008:2017)
> data.frame(Year,immig45.LPI=exp(immig45.LPI),log_back.predict,immig45.UPI=exp(immig45.UPI))

<table>
<thead>
<tr>
<th>Year</th>
<th>immig45.LPI</th>
<th>log_back.predict</th>
<th>immig45.UPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>200420.9</td>
<td>341907.9</td>
<td>546327.7</td>
</tr>
<tr>
<td>2009</td>
<td>151229.7</td>
<td>358462.9</td>
<td>724278.5</td>
</tr>
<tr>
<td>2010</td>
<td>121763.3</td>
<td>376970.0</td>
<td>899612.9</td>
</tr>
<tr>
<td>2011</td>
<td>101743.8</td>
<td>396680.0</td>
<td>1076638.7</td>
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<tr>
<td>2012</td>
<td>87043.3</td>
<td>417472.7</td>
<td>1258472.4</td>
</tr>
<tr>
<td>2013</td>
<td>75694.1</td>
<td>439366.3</td>
<td>1447161.9</td>
</tr>
<tr>
<td>2014</td>
<td>66627.2</td>
<td>462410.4</td>
<td>1644097.8</td>
</tr>
<tr>
<td>2015</td>
<td>59202.0</td>
<td>486663.7</td>
<td>1850302.3</td>
</tr>
<tr>
<td>2016</td>
<td>53005.9</td>
<td>512189.1</td>
<td>2066590.7</td>
</tr>
<tr>
<td>2017</td>
<td>47758.5</td>
<td>539053.3</td>
<td>2293659.5</td>
</tr>
</tbody>
</table>
Output-10: Forecasting IMA (1, 1) Model

```r
> immig45.log.ima_1.predict=predict(immig45.log.ima_1,n.ahead=10)
> round(immig45.log.ima_1.predict$pred,3)
Time Series:
Start = 2008
End = 2017
Frequency = 1
> round(immig45.log.ima_1.predict$se,3)
Time Series:
Start = 2008
End = 2017
Frequency = 1
[1] 0.256 0.398 0.501 0.587 0.661 0.728 0.790 0.846 0.900 0.950
> #Backtransromation
> log_back.ima_1.predict=round(exp(immig45.log.ima_1.predict$pred+(1/2)*(immig45.log.ima_1.predict$se)^2),3)
> #Prediction Interval
> immig45.log.ima_1.UPI=(immig45.log.ima_1.predict$pred)+(qnorm(0.975,0,1)*immig45.log.ima_1.predict$se)
> immig45.log.ima_1.LPI=(immig45.log.ima_1.predict$pred)-(qnorm(0.975,0,1)*immig45.log.ima_1.predict$se)
> Year=c(2008:2017)
> data.frame(Year,ima_1.LPI=exp(immig45.log.ima_1.LPI),log_back.ima_1.predict,ima_1.UPI=exp(immig45.log.ima_1.UPI))
Year ima_1.LPI log_back.ima_1.predict ima_1.UPI
1 2008 200510.16 342124.8 546753.8
2 2009 151737.82 358409.7 722494.1
3 2010 123906.78 375469.7 884775.5
4 2011 104800.40 393341.8 1046080.8
5 2012 90562.73 412064.5 1210538.7
6 2013 79430.75 431678.5 1380192.1
7 2014 70443.00 452226.0 1556289.3
8 2015 63016.15 473751.6 1739707.6
9 2016 56769.74 496301.8 1931129.0
10 2017 51442.20 519925.3 2131123.7
```
Output-11: Forecasting ARIMA (0, 1, 0) Model

```r
> immig45.log.random.walk.predict=predict(immig45.log.random.walk,n.ahead=10)
> round(immig45.log.random.walk.predict$pred,3)
Time Series:
Start = 2008
End = 2017
Frequency = 1
> round(immig45.log.random.walk.predict$se,3)
Time Series:
Start = 2008
End = 2017
Frequency = 1
[1] 0.261 0.369 0.452 0.522 0.584 0.640 0.691 0.739 0.783 0.826
> #Backtransormation
> log_back.random.walk.predict=round(exp(immig45.log.random.walk.predict$pred+(1/2)*(immig45.log.random.walk.predict$se)^2),3)
> #Prediction Interval
> immig45.random.walk.UPI=(immig45.log.random.walk.predict$pred)+(qnorm(0.975,0,1)*immig45.log.random.walk.predict$se)
> immig45.random.walk.LPI=(immig45.log.random.walk.predict$pred)-(qnorm(0.975,0,1)*immig45.log.random.walk.predict$se)
> Year=c(2008:2017)
> data.frame(Year,immig45.LPI=exp(immig45.random.walk.LPI),log_back.random.walk.predict,immig45.random.walk.UPI=exp(immig45.random.walk.UPI))
```

<table>
<thead>
<tr>
<th>Year</th>
<th>immig45.LPI</th>
<th>log_back.random.walk.predict</th>
<th>immig45.random.walk.UPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>198165.97</td>
<td>342091.7</td>
<td>551612.9</td>
</tr>
<tr>
<td>2009</td>
<td>160305.45</td>
<td>353959.3</td>
<td>681891.4</td>
</tr>
<tr>
<td>2010</td>
<td>136236.11</td>
<td>366238.7</td>
<td>802363.7</td>
</tr>
<tr>
<td>2011</td>
<td>118775.37</td>
<td>378944.0</td>
<td>920316.3</td>
</tr>
<tr>
<td>2012</td>
<td>105256.24</td>
<td>392090.0</td>
<td>1038521.8</td>
</tr>
<tr>
<td>2013</td>
<td>94363.41</td>
<td>405692.2</td>
<td>1158403.5</td>
</tr>
<tr>
<td>2014</td>
<td>85344.23</td>
<td>419766.2</td>
<td>1280823.6</td>
</tr>
<tr>
<td>2015</td>
<td>77725.74</td>
<td>434328.4</td>
<td>1406367.0</td>
</tr>
<tr>
<td>2016</td>
<td>71190.78</td>
<td>449395.9</td>
<td>1535464.5</td>
</tr>
<tr>
<td>2017</td>
<td>65516.24</td>
<td>464986.0</td>
<td>1668455.1</td>
</tr>
</tbody>
</table>
```