The heights of a certain population of corn plants follow a normal distribution with mean $\mu_Y = 145$ cm and standard deviation $\sigma_Y = 22$ cm.

(a) (4 points) What percentage of plants are between 140 cm and 150 cm tall?

Answer: This is

$$P(140 \leq Y_i \leq 150) = P\left(\frac{140 - 145}{22} \leq \frac{Y_i - 145}{22} \leq \frac{150 - 145}{22}\right)$$

$$= P(-0.227 \leq Z \leq 0.227)$$

$$\approx P(Z \leq 0.23) - P(Z \leq -0.23)$$

$$= 0.59 - 0.41$$

$$= 0.18.$$

Also: normalcdf(140,150,145,22).

(b) (4 points) Let $\bar{Y}$ be the mean height of a random sample of $n = 16$ plants from the population. What is $P(140 \leq \bar{Y} \leq 150)$?

Answer:

$$P(140 \leq \bar{Y} \leq 150) = P\left(\frac{140 - 145}{22/\sqrt{16}} \leq \frac{Y_i - 145}{22/\sqrt{16}} \leq \frac{150 - 145}{22/\sqrt{16}}\right)$$

$$= P(-0.91 \leq Z \leq 0.91)$$

$$\approx P(Z \leq 0.91) - P(Z \leq -0.91)$$

$$= 0.82 - 0.18$$

$$= 0.64.$$

Also: normalcdf(140,150,145,5.5).

(c) (4 points) What is $P(\bar{Y} \leq 145) = P(\bar{Y} \leq \mu_Y)$?

Answer: 0.5.