2.4 Discrete Probability Models

Recall: An ______ is the process by which an observation is made.

Defn: A sample point (or outcome) is the particular result of an experiment.

Defn: The sample space (denoted S) of an experiment is the set of all possible sample points.

Defn: A discrete sample space contains a finite or countable number of distinct sample points.

Example 1: Roll a fair die:

\[ S = \]

Example 2: Flip a fair coin until the first “head” and record the number of “tails” that have occurred.

\[ S = \]

Example 3: Randomly select an American household and record the number of TV sets.

\[ S = \]
Defn: An event is a collection of sample points (an event is a subset of S).

Note: We typically denote events by capital letters.

- A _____ event corresponds to exactly one sample point.
- A _____ event corresponds to more than one sample point.

Example 1: (Roll die)
Let Event $A = \{\text{roll an odd number}\}$
Let Event $B = \{\text{roll a 2}\}$

Example 2: Let event $C =$ "get head on first flip"
$C = \quad$
Let $D =$ "get head before third flip"
$D = \quad$

- We usually want to determine the probability of an event of interest.
- To do this, we can often assign probabilities to each sample point and then determine which sample point(s) correspond to the event.
3 Probability Axioms (Kolmogorov)

Let \( S \) be a sample space and \( A (\subset S) \) be an event. We assign \( P(A) \), the probability of \( A \), such that:

Axiom 1:

Axiom 2:

Axiom 3: If \( A_1, A_2, A_3, \ldots \) are pairwise mutually exclusive events in \( S \), then:

Corollary: If \( A_1, A_2, \ldots, A_n \) are pairwise mutually exclusive events, then

Proof:

Example 1: Let \( A_1 = \{1,3\}, A_2 = \{3,5\}, A_3 = \{5,7\} \).

\( P(A) = \)
Probability Rules following from the Axioms

1. Complement Rule: \( P(A) = 1 - P(\overline{A}) \).
   Proof:

2. \( P(\emptyset) = 0 \). Proof:

3. If \( A \subset B \), then \( P(A) \leq P(B) \).
   Proof:

Corollary: For any \( A \subset S \), \( P(A) \leq 1 \) (= \( P(S) \)).

Theorem (Additive Law of Probability):
\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
Proof:
Corollary: If A and B are mutually exclusive events, then

Note: For 3 events A, B, and C:

\[ P(A \cup B \cup C) = \]

Proof:
- In general, an extended additive law holds for any n events $A_1, A_2, \ldots, A_n$.

- In simple cases, we can list the sample points corresponding to an event and determine an appropriate probability for each sample point.

- To obtain the probability of the event $A$ in question, we sum the probabilities of the sample points in $A$.

Example 1 (a): Toss three coins, each weighted such that “heads” is twice as likely as “tails”.

Sample Space:
When all sample points in $S$ are equally likely, finding $P(A)$ is easier. In this case:

$$P(A) = \ldots$$

Example 1(b): Same experiment, except all three coins are fair.

Exercise: If we toss 10 fair coins, what is the probability of obtaining two or more “heads”? 