CHAPTER 2

2 Probability

Complementary reading: Chapter 2 (WMS).

2.1 Introduction

TERMINOLOGY: The text defines probability as a measure of one’s belief in the occurrence of a future (random) event. Probability is also known as “the mathematics of uncertainty.”

REAL LIFE EVENTS: Here are some events we may wish to assign probabilities to:

- tomorrow’s temperature exceeding 80 degrees
- getting a flat tire on my way home today
- a new policy holder making a claim in the next year
- the NASDAQ losing 5 percent of its value this week
- you being diagnosed with prostate/cervical cancer in the next 20 years.

ASSIGNING PROBABILITIES: How do we assign probabilities to events? There are three general approaches.

1. Subjective approach.
   - This approach is based on feeling and may not even be scientific.

2. Relative frequency approach.
   - This approach can be used when some random phenomenon is observed repeatedly under identical conditions.

3. Axiomatic/Model-based approach. This is the approach we will take in this course.

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\text{Ex: flip a coin.} \quad P(\text{head}) = \frac{1}{2}\]

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\lim_{n \to \infty} \frac{n(\text{head})}{n} \to \frac{1}{2}
\]
Figure 2.1: The relative frequency of die rolls which result in a “2”; each plot represents 1000 simulated rolls of a fair die.

Example 2.1. Relative frequency approach. Suppose that we roll a die 1000 times and record the number of times we observe a “2.” Let $A$ denote this event. The relative frequency approach says that

$$P(A) \approx \frac{\text{number of times } A \text{ occurs}}{\text{number of trials performed}} = \frac{n(A)}{n},$$

where $n(A)$ denotes the frequency of the event, and $n$ denotes the number of trials performed. The proportion $n(A)/n$ is called the relative frequency. The symbol $P(A)$ is shorthand for “the probability that $A$ occurs.”

**Relative Frequency Approach:** Continuing with our example, suppose that $n(A) = 158$. We would then estimate $P(A)$ by $158/1000 = 0.158$. If we performed the experiment of rolling a die repeatedly, the relative frequency approach says that

$$\frac{n(A)}{n} \rightarrow P(A),$$

as $n \rightarrow \infty$. Of course, if the die is fair, then $n(A)/n \rightarrow P(A) = 1/6$. □