STAT J540 Homework 3 – Fall 2012

1. (a) Using simulation techniques, write an R program to maximize the following function

$$h(x) = 0.25 + \cos(x)\left[\frac{1}{\sqrt{x}(1+\sqrt{x})^2} + 0.08\exp(-0.5(x-5)^2)\right]$$

over the interval: $x \in [2, 8]$.

(b) Plot this function in R as a smooth curve. Write commands to have R put text *within* your plot labeling the location of the maximum and the maximum value. Draw an arrow pointing at the peak where the maximum occurs.

(c) Using simulation techniques, evaluate the definite integral

$$\int_2^8 \left\{0.25 + \cos(x)\left[\frac{1}{\sqrt{x}(1+\sqrt{x})^2} + 0.08\exp(-0.5(x-5)^2)\right]\right\} dx.$$

For a variety of choices of n, try both the hit-and-miss method and the classical Monte Carlo integration method. Which one appears to converge faster to the eventual answer?

2. (a) Using simulation techniques, write an R program to maximize the following function

$$h(x_1, x_2) = 3 - 3x_1/\Big((x_1 - 0.2)^2 + x_2^2 + 1\Big)$$

over the region: $x_1 \in [-2, 2], x_2 \in [-2, 2]$.

Report both the maximum value of $h(x_1, x_2)$ and where (i.e., the (x_1, x_2) location) it occurs.

(b) Do a 3-D plot of this function of x_1 and x_2 (this response surface). Hint: Look carefully at the *first* example at the bottom of the help file for the `persp` function. You can adapt that example code to plot this function as a 3-D plot. Also: Try the `ticktype="detailed"` argument to the `persp` function.

(c) Using the simulation technique of your choice, evaluate the definite integral

$$\int_{-2}^{2}\int_{-2}^{2}\left\{3 - 3x_1/\Big((x_1 - 0.2)^2 + x_2^2 + 1\Big)\right\} dx_1 dx_2.$$

Use a variety of choices of n. Do the results appear to converge toward some answer?