Model: $Y_{ijk} = \mu_{\cdot\cdot} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$,
with $E(\varepsilon_{ijk}) = 0$.

- Let's compare levels 1 and 2 of factor A, i.e., estimate $\mu_{1\cdot} - \mu_{2\cdot} = (\mu_{\cdot\cdot} + \alpha_1) - (\mu_{\cdot\cdot} + \alpha_2) = \alpha_1 - \alpha_2$
- Note: Let's assume no interaction between A and B so that this direct comparison is appropriate.
- So $(\alpha\beta)_{ij} = 0$ for all i, j.

A \ B	1	2	3
1	x x x	x x	x x
2	x	x x x	x x x

How to estimate $\mu_{1\cdot}$ and $\mu_{2\cdot}$?

LSMEANS approach (use unweighted average of cell sample means):

$$\left(\tfrac{1}{3}\bar{Y}_{11\cdot} + \tfrac{1}{3}\bar{Y}_{12\cdot} + \tfrac{1}{3}\bar{Y}_{13\cdot}\right) - \left(\tfrac{1}{3}\bar{Y}_{21\cdot} + \tfrac{1}{3}\bar{Y}_{22\cdot} + \tfrac{1}{3}\bar{Y}_{23\cdot}\right)$$

has expected value

$$\tfrac{1}{3}(\mu_{\cdot\cdot} + \alpha_1 + \beta_1) + \tfrac{1}{3}(\mu_{\cdot\cdot} + \alpha_1 + \beta_2) + \tfrac{1}{3}(\mu_{\cdot\cdot} + \alpha_1 + \beta_3)$$
$$- \tfrac{1}{3}(\mu_{\cdot\cdot} + \alpha_2 + \beta_1) - \tfrac{1}{3}(\mu_{\cdot\cdot} + \alpha_2 + \beta_2) - \tfrac{1}{3}(\mu_{\cdot\cdot} + \alpha_2 + \beta_3)$$
$$= \alpha_1 - \alpha_2$$

MEANS approach (use row sample means):

$\bar{Y}_{1\cdot\cdot} - \bar{Y}_{2\cdot\cdot}$, for these data, is:

$$\left(\tfrac{3}{7}\bar{Y}_{11\cdot} + \tfrac{2}{7}\bar{Y}_{12\cdot} + \tfrac{2}{7}\bar{Y}_{13\cdot}\right) - \left(\tfrac{1}{7}\bar{Y}_{21\cdot} + \tfrac{3}{7}\bar{Y}_{22\cdot} + \tfrac{3}{7}\bar{Y}_{23\cdot}\right)$$

$$\Rightarrow \tfrac{3}{7}(\mu_{\cdot\cdot} + \alpha_1 + \beta_1) + \tfrac{2}{7}(\mu_{\cdot\cdot} + \alpha_1 + \beta_2) + \tfrac{2}{7}(\mu_{\cdot\cdot} + \alpha_1 + \beta_3)$$
$$- \tfrac{1}{7}(\mu_{\cdot\cdot} + \alpha_2 + \beta_1) - \tfrac{3}{7}(\mu_{\cdot\cdot} + \alpha_2 + \beta_2) - \tfrac{3}{7}(\mu_{\cdot\cdot} + \alpha_2 + \beta_3)$$
$$= \alpha_1 - \alpha_2 + \tfrac{2}{7}\beta_1 - \tfrac{1}{7}\beta_2 - \tfrac{1}{7}\beta_3$$