STAT 535 – Spring 2022 – HW 2

1. Do Problem 4.4 in the Chapter 4 Exercises of the *Bayes Rules!* text.

2. (Graduate students only; extra credit for undergrads): Do Problem 4.10 in the Chapter 4 Exercises of the *Bayes Rules!* text.

3. Do Problem 4.16 in the Chapter 4 Exercises of the *Bayes Rules!* text.

4. Do Problem 4.19 in the Chapter 4 Exercises of the *Bayes Rules!* text. Hint: The code

```
bechdel %>% filter(year==1980)
```

will pick out the movies in the data set from the year 1980.

5. The eBay selling prices for auctioned Palm M515 PDAs are assumed to follow a normal distribution with $\mu$ and $\sigma^2$ unknown. We wish to perform inference on the mean selling price $\mu$.

(a) Suppose we assume an $IG(1100, 250000)$ prior for $\sigma^2$ and let the prior for $\mu|\sigma^2$ be

$$p(\mu|\sigma^2) \propto (\sigma^2)^{-1/2} e^{-\frac{1}{2\sigma^2/s_0}(\mu-\delta)^2},$$

with $s_0 = 1$ and $\delta = 220$. If our sample data are: (212, 249, 250, 240, 210, 234, 195, 199, 222, 213, 233, 251), then find a point estimate and 95% credible interval for $\mu$.

(b) Now suppose (perhaps unrealistically) that we had known the true population variance was $\sigma^2 = 228$. Assuming a conjugate prior for $\mu$ with $\delta = 220$ and $\tau^2 = 25$, find a point estimate and 95% credible interval for the single unknown parameter $\mu$.

(c) How (if at all) does the inference in part (b) differ from the inferences in part (a)? Explain your answer intuitively.

6. Do Problem 5.5 in the Chapter 5 Exercises of the *Bayes Rules!* text.

7. Do Problem 5.6 in the Chapter 5 Exercises of the *Bayes Rules!* text.

8. Do Problem 5.12 in the Chapter 5 Exercises of the *Bayes Rules!* text. [Hint: For part (a), use `filter(group == "control")` to pick out the "control" subjects.]

9. Do Problem 5.19 in the Chapter 5 Exercises of the *Bayes Rules!* text.

10. A researcher is trying to estimate the mean number of accidents per month within 100 feet of the Gervais Street/Assembly Street intersection in Columbia. She assumes a Poisson($\lambda$) model for the number of accidents $Y$ per month, so that the density function for $Y$ given $\lambda$ is

$$p(y|\lambda) = \frac{\lambda^y e^{-\lambda}}{y!}, \quad y = 0, 1, 2, \ldots, \ \lambda \geq 0.$$

(a) She uses a standard exponential prior distribution for $\lambda$ (i.e., an exponential with mean 1, which is the same as a gamma distribution with shape 1 and rate 1). Derive the general form of her posterior distribution for $\lambda$ given a random sample $y_1, \ldots, y_n$ from $n$ weeks.

(b) If she gathers the following accident counts from 15 randomly selected months

```
1 0 4 1 4 2 5 3 0 3 1 2 2 4 1
```

find the posterior mean and a 95% credible interval (get both a quantile-based interval and a HPD interval) for $\lambda$ using the standard exponential prior, along with these data.