STAT 509 – Section 7.3: More Experimental Design

• Unless k is quite small, full 2^k factorial experiments require many experimental runs.

• <u>Fractional factorial</u> experiments are designed to reduce the required number of runs while maintaining the factorial structure and the ability to examine main effects and interaction effects of interest.

• Fractional factorials do this by reducing the number of treatment combinations examined, and thus forgoing the ability to estimate "higher-order" interactions.

• In most experiments, the high-order interactions (interactions among several factors) are not as important as the main effects and low-order (such as two-factor) interactions.

Example: Half Fraction of a 2^3 Design

• A full 2^3 factional experiment requires (even in the case of no replication) experimental runs.

• In situations where experimental runs are time-consuming or costly, we may wish to obtain good conclusions with fewer than 2^k runs.

Table of Contrasts for a Full 2^3 Factorial Design

I	x1	x2	x3	x1x2	x1x3	x2x3	x1x2x3
1	-1	-1	-1	1	1	1	-1
1	1	-1	-1	-1	-1	1	1
1	-1	1	-1	-1	1	-1	1
1	1	1	-1	1	-1	-1	-1
1	-1	-1	1	1	-1	-1	1
1	1	-1	1	-1	1	-1	-1
1	-1	1	1	-1	-1	1	-1
1	1	1	1	1	1	1	1

• Suppose we remove all the rows in which the column `x1x2x3` has `-1`. This leaves us with:

I	x1	x2	x3	x1x2	x1x3	x2x3	x1x2x3
1	1	-1	-1	-1	-1	1	1
1	-1	1	-1	-1	1	-1	1
1	-1	-1	1	1	-1	-1	1
1	1	1	1	1	1	1	1

• Advantage: We are down to four rows, meaning we need only four experimental runs.

• Disadvantage: The column for `I` and the column for `x1x2x3` are exactly the same. This implies we cannot estimate both the intercept and the three-factor interaction effect.

• We say the three-factor interaction, ABC, is <u>aliased</u> with the intercept.

• In addition: The columns for and for are exactly the same.

• So the main effect for factor A is <u>aliased</u> with the two-factor interaction BC.

• Similarly, the main effect for factor B is <u>aliased</u> with the two-factor interaction

• And the main effect for factor C is <u>aliased</u> with the two-factor interaction

• So in this half-fraction design, we *cannot distinguish* the main effect of any one factor from the interaction effect of the other two factors.

• Only solution? Use a model that assumes the interactions are unimportant:

<u>Linear Model for the 2^{3-1} Factorial Design</u>

$$Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \varepsilon_i$$

The notation "2^{3-1} Factorial" indicates there are 2 levels for each factor; there are 3 factors, and it is a half fraction.

• The total number of treatment combinations is 2^{3-1} =

• If the interactions are indeed unimportant, this model is fine.
• If we use this half-fraction model and we *do* have important interactions, we can make false conclusions: We might mistakenly conclude a main effect is significant when it actually is not.

• In this example, ABC is called the <u>defining interaction</u> because we picked a specific level for `x1x2x3` to select which treatment combinations to run.

<u>Determining the Alias Structure</u>

• We can quickly determine which factors are aliased in the following way:
• The highest-order interaction is the defining interaction and is equated to the intercept, I.
• We add each effect to the defining interaction using modulo 2 arithmetic (where 1 + 1 = 0).
• For example, in the 2^{3-1} design:

A Real Data Example with Four Factors

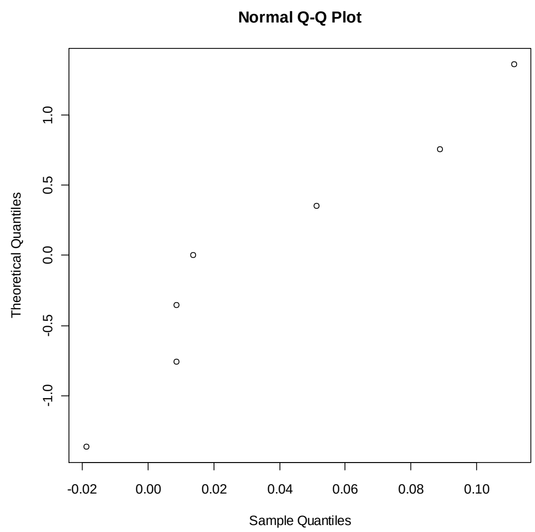
• **Table 7.44 gives the experimental results from a fractional factorial with a response variable Y = *free height* of a leaf spring, and 4 factors related to the heating process:**

- **High-heat temp. (x_1): 1840, 1880**
- **Heating time (x_2): 23, 25**
- **Transfer time (x_3): 10, 12**
- **Hold-down time (x_4): 2, 3**

Determining the Alias Structure for a 2^{4-1} Design here:

R code:

```
> leaf.data <- read.table(file =
"http://www.stat.sc.edu/~hitchcock/leafspringdata.txt",
header=T)
> attach(leaf.data)
> summary(lm(y ~ x1 * x2 * x3 * x4))
> qqnorm(coef(lm(y ~ x1 * x2 * x3 * x4))[-1],datax=T)
```



• Based on the magnitudes of the estimated coefficients and the normal Q-Q plot of the estimated coefficients, which effects appear to be significant?

<u>Final Comments on Experimental Design</u>

• Some experimenters use a "one-factor-at-a-time" (OFAAT) approach to designing experiments.
• This consists of an initial run in which all factors are set to the same level (say, "low") and subsequent runs in which one factor at a time is changed from low to high:

• This approach has serious disadvantages compared to factorial (or fractional factorial) designs:
(1) The OFAAT approach cannot estimate interactions.
(2) The OFAAT approach does not examine the entire experimental region of treatment combinations.
(3) The effect estimates resulting from a OFAAT approach are <u>not as precise</u> as the estimates from a factorial (or fractional factorial) design.

• Other experimenters use a "shotgun" approach to design, in which they select treatment combinations randomly over the experimental region.
• This approach is also not preferred, since it tends to waste resources, miss important parts of the experimental region, and/or produce less precise estimates of effects.