STAT 515 -- Chapter 4: Discrete Random Variables

Random Variable: A variable whose value is the numerical outcome of an experiment or random phenomenon.

Discrete Random Variable : A numerical r.v. that takes on a countable number of values (there are gaps in the range of possible values).

Examples:

1. Number of phone calls received in a day by a company
2. Number of heads in 5 tosses of a coin

Continuous Random Variable : A numerical r.v. that takes on an uncountable number of values (possible values lie in an unbroken interval).

Examples:

1. Length of nails produced at a factory
2. Time in 100-meter dash for runners

Other examples?

The **probability distribution** of a random variable is a graph, table, or formula which tells what values the r.v. can take and the probability that it takes each of those values.

Example 1: Roll 1 die. The r.v. X = number of dots showing.

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

Example 2: Toss 2 coins. The r.v. X = number of heads showing.

x	0	1	2
P(x)	¼	½	¼

Graph for Example 2:

For <u>any</u> probability distribution:

(1) P(x) is between 0 and 1 for any value of x.

(2) $\sum_x P(x) = 1$. That is, the sum of the probabilities for all possible x values is 1.

Example 3: P(x) = x / 10 for x = 1, 2, 3, 4.

Valid Probability Distribution?
Property 1?
Property 2?

Expected Value of a Discrete Random Variable

The expected value of a r.v. is its mean (i.e., the mean of its probability distribution).

For a discrete r.v. X, the expected value of X, denoted μ or $E(X)$, is:

$$\mu = E(X) = \Sigma\, x\, P(x)$$

where Σ represents a summation over all values of x.

Recall Example 3:

$\mu =$

Here, the expected value of X is

Example 4: Suppose a raffle ticket costs \$1. Two tickets will win prizes: First prize = \$500 and second prize = \$300. Suppose 1500 tickets are sold. What is the expected profit for a ticket buyer?

x (profit)	
$P(x)$	

$E(X) =$

$E(X) = -0.47$ dollars, so on average, a ticket buyer will lose 47 cents.

The expected value does not have to be a possible value of the r.v. --- it's an average value.

Variance of a Discrete Random Variable

The variance σ^2 is the expected value of the squared deviations from the mean μ; that is, $\sigma^2 = E[(X - \mu)^2]$.

$$\sigma^2 = \Sigma (x - \mu)^2 P(x)$$

Shortcut formula:

$$\sigma^2 = [\Sigma x^2 P(x)] - \mu^2$$

where Σ represents a summation over all values of x.

Example 3: Recall $\mu = 3$ for this r.v.

$\Sigma x^2 P(x) =$

Thus $\sigma^2 =$

Note that the standard deviation σ of the r.v. is the square root of σ^2.

For Example 3, $\sigma =$

The Binomial Random Variable

Many experiments have responses with 2 possibilities (Yes/No, Pass/Fail).

Certain experiments called binomial experiments yield a type of r.v. called a binomial random variable.

Characteristics of a binomial experiment:

(1) The experiment consists of a number (denoted n) of identical trials.

(2) There are only two possible outcomes for each trial – denoted "Success" (S) or "Failure" (F)

(3) The probability of success (denoted p) is the same for each trial.
(Probability of failure = $q = 1 - p$.)

(4) The trials are independent.

Then the binomial r.v. (denoted X) is the number of successes in the n trials.

Example 1: A fair coin is flipped 5 times. Define "success" as "head". X = total number of heads. Then X is

Example 2: A student randomly guesses answers on a multiple choice test with 3 questions, each with 4 possible answers. X = number of correct answers. Then X is

What is the probability distribution for *X* in this case?

Outcome	***X***	**P(outcome)**

Probability Distribution of *X*

x	**P(*x*)**

<u>General Formula</u>: (Binomial Probability Distribution)
(n = number of trials, p = probability of success.)
The probability there will be exactly x successes is:

$$P(x) = \binom{n}{x} p^{x} q^{n-x} \quad (x = 0, 1, 2, \ldots, n)$$

where

$$\binom{n}{x} = \text{“}n \text{ choose } x\text{”}$$

$$= \frac{n!}{x!\,(n-x)!}$$

Here, $0! = 1$, $1! = 1$, $2! = 2\cdot 1 = 2$, $3! = 3\cdot 2\cdot 1 = 6$, etc.

Example: Suppose probability of “red” in a roulette wheel spin is 18/38. In 5 spins of the wheel, what is the probability of exactly 4 red outcomes?

- **The mean (expected value) of a binomial r.v. is $\mu = np$.**
- **The variance of a binomial r.v. is $\sigma^2 = npq$.**
- **The standard deviation of a binomial r.v. is $\sigma =$**

Example: What is the mean number of red outcomes that we would expect in 5 spins of a roulette wheel?

$\mu = np =$

What is the standard deviation of this binomial r.v.?

Using Binomial Tables

Since hand calculations of binomial probabilities are tedious, Table II gives "cumulative probabilities" for certain values of n and p.

Example:
Suppose X is a binomial r.v. with $n = 10$, $p = 0.40$.
Table II (page 886) gives:

Probability of 5 or fewer successes: $P(X \leq 5) =$

Probability of 8 or fewer successes: $P(X \leq 8) =$

What about …

… the probability of exactly 5 successes?

… the probability of more than 5 successes?

… the probability of 5 or more successes?

… the probability of 6, 7, or 8 successes?

Why doesn't the table give $P(X \leq 10)$?

Poisson Random Variables

The Poisson distribution is a common distribution used to model "count" data:

- **Number of telephone calls received per hour**
- **Number of claims received per day by an insurance company**
- **Number of accidents per month at an intersection**

The mean number of events for a Poisson distribution is denoted λ.

Which values can a Poisson r.v. take?

Probability distribution for *X*
(if *X* is Poisson with mean λ)

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} \qquad (\text{for } x = 0, 1, 2, \ldots)$$

Mean of Poisson probability distribution: λ

Variance of Poisson probability distribution: λ

Example: A call center averages 10 calls per hour. Assume *X* (the number of calls in an hour) follows a Poisson distribution. What is the probability that the call center receives exactly 3 calls in the next hour?

What is the probability the call center will receive 2 or more calls in the next hour?

Calculating Poisson probabilities by hand can be tedious. Table III gives cumulative probabilities for a Poisson r.v., P($X \leq k$) for various values of k and λ.

Example 1: X is Poisson with $\lambda = 1$. Then

P($X \leq 1$) =

P($X \geq 3$) =

P($X = 2$) =

Example 2: X is Poisson with $\lambda = 6$. Then

… probability that X is 5 or more?

… probability that X is 7, 8, or 9?