<u>Two-Way Tables</u>

• The simplest two-way table is a 2 × 2 table, with 2 categorical variables, each with 2 categories.

• The analysis of 2 × 2 tables greatly depends on how the samples were taken from the population.

Inference about 2 Proportions – Two Independent Samples

• Consider taking independent samples (size n_1 and n_2) from two populations and observing <u>the same</u> binary variable for each.

Example: Survey of 160 rural households and 261 urban households, each with one Christmas tree. Observe whether their tree is natural or artificial.

2 × 2 table:

Model:

Again,

Example above:

__Of interest__: Estimate $\pi_1 - \pi_2$ (with a CI) or test H_0: $\pi_1 = \pi_2$ (or possibly H_0: $\pi_1 - \pi_2 = \Delta_0$).

Again, LS (and ML) estimator of π_i (i = 1, 2) is:

so LS (and ML) estimator of $\pi_1 - \pi_2$ is:

And by CLT, for large samples,

Again, for large n_1 and n_2, a $100(1 - \alpha)\%$ CI for $\pi_1 - \pi_2$ is:

Test statistic for a test of H_0: $\pi_1 = \pi_2$ is:

(Under H_0, $\pi_1 = \pi_2 = \pi$, which we estimate by

If n_1 and n_2 large, rejection rules are:

Rule of thumb: Large-sample methods are appropriate if the number of successes and number of failures in each sample is at least 5.

(If

Example (Christmas trees). Let rural = population 1, urban = population 2.

Small-sample alternative: Fisher's Exact Test

• If we have two independent samples that are too small to use the z-test, we can use Fisher's exact test to test H_0: $\pi_1 = \pi_2$.

• H_a could be $\pi_1 < \pi_2$, $\pi_1 > \pi_2$, or $\pi_1 \neq \pi_2$.

• We use ________________________ probabilities to calculate the P-value.

• Suppose our observed table is:

Notation here:

• **The test assumes (1) H_0: $\pi_1 = \pi_2$, and (2) out of $n_1 + n_2$ observations, we have m_1 overall "successes".**

• **Given (1) and (2), our P-value is the probability of observing cell counts at least as favorable to H_a as the counts that we actually observed.**

Example: If H_a: $\pi_1 < \pi_2$, then our P-value is

• **Note: Fisher's test is exact if the numbers in both of the margins of the table are truly fixed by the sampling scheme.**
• **This occurs in some specialized situations.**

• **Fisher's test is approximate if the margins are not fixed.**

Example: Six forest sites are randomly sampled in each of two North Florida counties and each site is labeled "mainly pine" or "mainly non-pine".
• **The data are summarized:**

• Are both margins fixed here?

• We test whether the first county has a smaller true proportion of pine sites than the second county:

P-value =

<u>R example</u> (`fisher.test` function):

<u>Note</u>: Fisher's Exact test tends to have _________ power for small samples.

• R also reports a CI for the "odds ratio":

<u>Comparing two proportions: Paired Samples</u>

• Sometimes the observations in two samples are <u>not</u> <u>independent</u>.

• We may have (1) two binary measurements on the same subject, or (2) binary measurements on subjects that are naturally paired.

<u>Example 1</u>: 60 couples (husbands and wives) are surveyed about their marriage.
<u>Response</u>: Are you satisfied in your marriage or not?
<u>Of interest</u>: Is the proportion of satisfied people different for husbands than for wives?

<u>Example 2</u>: 200 customers were given two brands (name-brand and economy) of vanilla ice cream in a random order.
• Each ice cream was rated as "like" or "did not like" by each customer.
• Since the same subject was used twice, the observations are naturally paired:

Data:

<u>Question</u>: Is the proportion of customers who like the name brand different from the proportion who like the economy brand?

• That is, compare π_1 to π_2.

<u>McNemar's Test</u>

Denote the observed table by:

- **Let $\hat{\pi}_1$ = proportion of "successes" in Sample 1.**
- **Let $\hat{\pi}_2$ = proportion of "successes" in Sample 2.**

It can be shown (under H_0):

Testing H_0: $\pi_1 = \pi_2$:

For large *n*, under H_0,

H_a **Reject H_0 if:**

• An approximate $(1 - \alpha)100\%$ CI for $\pi_1 - \pi_2$ is:

<u>Rule of Thumb</u>: These large-sample procedures are valid if:

<u>R example</u> (ice cream data) (the `mcnemar.test` function only does the two-sided test):

95% CI for $\pi_1 - \pi_2$:

<u>Note</u>: Exact tests have been developed for this type of inference when the sample is small.

• One option is to do a sign test (recall from STAT 704) where:

T_{12} is treated as the number of "positive differences" and $T_{12} + T_{21}$ is treated as the "total number of observations".

• If T_{12} is an "unusual" value relative to the Binomial($T_{12} + T_{21}$, 0.5) distribution, this is evidence against H_0.