Binary Probit Regression

- Note that if Y is binary (two-category), the same model could hold, with $K = 2$.
- So we have only one threshold g_1 separating the two categories.
- **Example 2** (54 elderly patients): Let

$$Y_i = \begin{cases} 1 & \text{if senility is not present in individual } i \\ 2 & \text{if senility is present in individual } i \end{cases}$$

- Explanatory variable X = score on subset of WAIS intelligence test.
- See R example on course web page.

Bayesian Logistic Regression

- The **logistic regression** approach does not assume the unobserved latent variable is normally distributed.
- We define Y to be either 1 (success) or 0 (failure) and model $P(Y = 1)$ at a given value x of the explanatory variable X as:

$$\pi_x = \frac{e^{\beta_0+\beta_1 x}}{1 + e^{\beta_0+\beta_1 x}}$$

where $\pi_x = P(Y = 1|X = x)$.

- This is equivalent to modeling the **log-odds** of success with a linear predictor function:

$$logit(\pi_x) = \ln\left(\frac{\pi_x}{1 - \pi_x}\right) = \beta_0 + \beta_1 x$$

Bayesian Logistic Regression

- A common choice is choosing normal priors on the regression coefficients (β_0 and β_1).
- Alternately, we could specify beta priors on the success probabilities at selected x-values of interest.
- We could then express the β's deterministically in terms of those success probabilities by back-solving.
- See `WinBUGS` examples on course web page with senility data.