Probability Models

- **Example 1: If we pick a person at random, what are the possible blood types of that person? Which blood type is the likeliest?**
- **A *probability model* presents a *probability distribution* for some random phenomenon.**
- **It gives (1) the possible outcomes of the random phenomenon, and for each outcome, (2) the probability of that outcome occurring.**
- **Often probability models are used to approximate real-world data processes.**

Probability Models: An Example

Example 1: If we pick a person at random, what are the possible blood classifications of that person?

Table 1: Probability distribution of blood classifications (including type and Rh factor).

Blood Class	Probability
O+	0.37
A+	0.34
B+	0.10
AB+	0.04
O-	0.06
A-	0.06
B-	0.02
AB-	0.01

Probability Terms

- **An *outcome* is the result of some random phenomenon or experiment.**
- **The *sample space* is the listing of all the possible outcomes of a random experiment.**
- **An *event* is some outcome or collection of outcomes of interest.**
- **Two events are *mutually exclusive* if they have no outcomes in common (thus they cannot occur simultaneously).**
- **Example 1 (Rolling a 6-sided die): Event A = $\{$Rolling a 1$\}$**
  Event B = $\{$Rolling an even number$\}$ Mutually exclusive?

Probability Rules

- **All probability distributions must follow certain mathematical rules.**

1. **Any probability is a number between 0 and 1.**
2. **The probabilities of all the outcomes in a distribution must add to 1.**
3. **The probability that some random event *does not occur* is 1 minus the probability that the event *does occur*.**
4. **If two events are *mutually exclusive*, then the probability that one or the other of the events occurs is the sum of the separate probabilities for the two individual events.**

Probability Coherence

- **A probability model that follows Rules 1 and 2 is called *legitimate* or *valid*.**
- **The probability model may be *wrong* (it may not reflect reality) – but it is mathematically *coherent*.**
- **Note: If Rules 1 and 2 are true, then Rules 3 and 4 will automatically be true.**
- **If a probability model does not follow Rules 1 and/or 2, it is called *incoherent*.**
- **Table 18.1 (p. 424) gives an incoherent set of probabilities: The probabilities add up to 1.46.**

Example with Probability Rules

- **A person with A+ blood may safely receive the following types of blood: O+, A+, O-, A-.**
- **Jerry has A+ blood and needs a transfusion. If a person is randomly selected to give blood, what is the probability that this person could safely donate blood to Jerry?**

Example with Probability Rules

- **A person with A+ blood may safely receive the following types of blood: O+, A+, O-, A-.**
- **Jerry has A+ blood and needs a transfusion. What is the probability that a randomly selected person could safely donate blood to Jerry?**
- **P(Jerry can receive the blood) = P(person has O+, A+, O-, *or* A- blood).**
- **Since these are *mutually exclusive*, this is**

  P(O+) + P(A+) + P(O-) + P(A-) = 0.37 + 0.34 + 0.06 + 0.06 = 0.83.
- **Jerry has an 83% chance of being able to receive the blood of the randomly selected person.**

Another Example with Probability Rules

- **A person with A- blood may safely receive the following types of blood: O-, A-.**
- **Jane has A- blood and needs a transfusion. If a person is randomly selected to give blood, what is the probability that this person could NOT safely donate blood to Jane?**
- **P(Jane cannot receive the blood) = 1 - P(Jane can receive the blood)**
- **Note: P(Jane can receive the blood) = 0.06 + 0.06 = 0.12.**
- **So P(Jane cannot receive the blood) = 1 - 0.12 = 0.88**
- **Jane has an 88% chance of NOT being able to receive the blood of the randomly selected person.**

Clicker Quiz 1

In gratitude, Jerry (who has A+ blood) decides to donate blood for others. Note that A+ blood can only be safely *given to* patients who have A+ blood or AB+ blood. If a patient is randomly selected, what is the probability that he can receive Jerry's blood?

A. 0.34

B. 0.38

C. 0.62

D. 0.04

Clicker Quiz 2

In gratitude, Jerry (who has A+ blood) decides to donate blood for others. Note that A+ blood can only be safely *given to* patients who have A+ blood or AB+ blood. If a patient is randomly selected, what is the probability that he CANNOT receive Jerry's blood?

A. 0.34

B. 0.38

C. 0.62

D. 0.04

Sampling Distributions

- **Recall that a *statistic* is a number that can be calculated from sample data.**
- **Examples of statistics: sample proportion, sample mean, sample median.**
- **Imagine taking repeated samples of a certain size (from the same population) and calculating a particular statistic each time.**

Sampling Distributions (Continued)

- **A *sampling distribution* describes the distribution of values of the statistic across many repeated samples.**
- **What values can the statistic take? How often would we expect it to take those values?**
- **For many statistics, the sampling distribution can be described by a density curve (specifically, the normal density curve).**

Example of a Sampling Distribution

- **Suppose that 10% of all 8th-graders in the U.S. have used marijuana at least once.**
- **Consider taking a random sample of 100 8th-graders and finding the sample proportion that have used marijuana.**
- **If we take many such samples and calculate the sample proportion $\hat{p}$ each time, what will be the *pattern* of $\hat{p}$ values?**
- **The *sampling distribution* for the sample proportion here will be an approximately normal distribution, with mean 0.10 and standard deviation 0.03. (see example applet)**

Example of a Sampling Distribution (Continued)

- **Suppose that 10% of all 8th-graders in the U.S. have used marijuana at least once.**
- **Consider taking a random sample of 100 8th-graders and finding the sample proportion that have used marijuana.**
- **The *sampling distribution* for the sample proportion here will be an approximately normal distribution, with mean 0.10 and standard deviation 0.03.**
- **If we take such a random sample, what is the probability that the sample proportion will be between 0.10 and 0.13?**
- **If we take such a random sample, what is the probability that the sample proportion will be greater than 0.19?**

Clicker Quiz 3

In the previous example, if we take random sample of 100 8th-graders and find the sample proportion that have used marijuana, what is the probability that the sample proportion will be between 0.10 and 0.16?

A. 0.68

B. 0.95

C. 0.34

D. 0.475