Simulation

- **Example 1: What is the probability of getting dealt a "full house" hand in poker?**
- **We can set up a *probability model* for the random experiment of being dealt five playing cards at random.**
- **But the model is very complicated, and there are many outcomes that lead to a "full house."**
- **It may be difficult to calculate the probability of a "full house" mathematically.**
- **Often it is easier to use computer *simulation* to estimate this probability.**

Simulation Example

- **We could write a computer program that would mimic the random dealing of five cards.**
- **The computer program could do the 5-card deals *many times*.**
- **We could keep track of the number of times we get a “full house”.**
- **The *estimated probability* of drawing a full house would be the number of full house results divided by the overall number of deals.**
- **Often the computer programs use *random digits* to “perform” the random experiments.**

Simple Example

- **Steph Curry of the Golden State Warriors is a 90% free throw shooter.**
- **If Curry takes 30 free throws in succession, what is the probability that he will miss three in a row some time in that series?**
- **Could calculate this mathematically, but it would be hard!**
- **How could we estimate this probability via *simulation*?**

Simple Example (Continued)

- **If Steph Curry (a 90% free throw shooter) takes 30 free throws in succession, what is the probability that he will miss three in a row some time in that series?**
- **“Simulate” 30 independent trials of successes and failures.**
- **Check whether 3 failures in a row occurred during those 30 trials.**
- **Repeat this *many* times, keeping track of how many times such a “bad streak” happened in the series.**
- **The probability we want is the number of series containing such a bad streak, divided by the overall number of series we generated.**

Simple Example (Continued more)

- **We could use a table of random numbers to do this: Let the digits 0 though 8 represent "makes" and the digit 9 represent a "miss".**
- **Continually look at sequences of 30 random digits. What proportion of these 30-digit sequences contain three 9's in a row?**
- ***Related Question*: How could we do the same experiment using the random digit table if Curry were a 95% free throw shooter?**
- **In practice using the table would be tedious – a computer can do this much more quickly!**

Clicker Quiz 1

Suppose the probability of having a boy for a random birth is 0.51. How could we use a random number table to estimate the probability of having two boys and two girls, in a random family of four kids?

A. Assign two-digit sequences to represent possible genders. Repeatedly examine sequences of eight digits in the table.

B. Assign one-digit sequences to represent possible genders. Repeatedly examine sequences of eight digits in the table.

C. Assign two-digit sequences to represent possible genders. Repeatedly examine sequences of four digits in the table.

D. Assign one-digit sequences to represent possible genders. Repeatedly examine sequences of four digits in the table.

Clicker Quiz 2

Suppose the probability of having a boy is 0.5. What would be the simplest way to use a random number table to estimate the probability of having two boys and two girls, in a random family of four kids?

A. Assign two-digit sequences to represent possible genders. Repeatedly examine sequences of eight digits in the table.

B. Assign one-digit sequences to represent possible genders. Repeatedly examine sequences of eight digits in the table.

C. Assign two-digit sequences to represent possible genders. Repeatedly examine sequences of four digits in the table.

D. Assign one-digit sequences to represent possible genders. Repeatedly examine sequences of four digits in the table.

Family Example on the Computer

- **Tools are available online for simple examples of simulations with these basic probability models.**
- **See applet for examining the number of girls in a 3-child family.**
- ***Note:* This program assumes the probability of having a boy is 0.5.**
- **What is the estimated probability of having no girls in a random 3-child family?**
- **What is the estimated probability of having 1 girl in a random 3-child family?**

Clicker Quiz 3

Suppose a mother has two children (both boys). If she ends up with a 3-child family, what is the probability that she has no girls among the children? (Note that this is a *conditional probability*: the probability of zero girls *given that* the first two children are boys.)

A. Near 0.125

B. Near 0.5

C. Near 0

D. Near 0.875

Independence

- **The simulation examples we've looked at assume the different random trials are *independent*.**
- ***Independent* trials means that the outcome of any one trial doesn't affect the probabilities for any other trial.**
- **Note that the answer to Clicker Quiz 3 assumed the trials were independent.**
- **For the rolling a die or tossing a coin, this makes sense.**
- **For other random phenomena, we could use sample data to verify whether the *assumption of independence* is reasonable. (Free throws? Baby genders?)**

Independence (Continued)

- **If the trials are *independent*, then there should be *near zero correlation* across observations.**
- **Studies have indicated that basketball shooting results can be considered independent.**
- **Some studies have cast doubt about whether baby genders are independent within a family.**

Tree Diagrams and Dependent Events

- ***Example:*** **A patient with kidney failure is waiting for a transplant.**
- **90% of patients survive the transplant operation and 10% die from the operation.**
- **Of those who survive the operation, the new kidney is successful in 60% of those, but the other 40% must return to dialysis.**
- **Among the patients with a successful new kidney, 70% will survive for at least five years.**
- **Among the patients returning to dialysis, 50% will survive for at least five years.**
- **For the original patient (who is waiting for the new kidney transplant), what is the probability of surviving at least five years?**

More on the Kidney Transplant Survival Example

- **For the original patient (who is waiting for the new kidney transplant), what is the probability of surviving at least five years?**
- **The information in this situation can be organized using a tree diagram (see picture).**
- **At each stage of the diagram, the outcomes can be simulated.**

Still More on the Kidney Transplant Survival Example

- **Note that the probability of surviving at least five years is *different* depending on the second stage (whether the new kidney is successful).**

- **The five-year survival chances *depend on* the success of the new kidney.**

- **The stages of this phenomenon are therefore *dependent*.**

- **A careful analysis reveals the probability of the patient surviving at least 5 years is 0.558.**

Clicker Quiz 4

***Given that the patient survives the operation*, what is his chance of surviving at least five years?**

A. 0.558

B. Less than 0.558

C. More than 0.558

D. We cannot say for sure which of the above choices is true.