Expected Values

- **Example 1: A campus organization is holding a raffle to raise money. There are two prizes: a $200 gift certificate to the campus bookstore, and a $50 gift certificate.**
- **1000 raffle tickets will be sold (at $1 apiece), and two of the tickets will be winners.**
- **For a ticket buyer, what is the expected return?**
- **This would indicate what a buyer would consider a “fair price” (disregarding the philanthropic aspect!)**

Calculating Expected Values

- **We can easily calculate the *expected value* of a random phenomenon that has a finite number of possible numerical outcomes.**
- **First we must specify a probability model giving the probability of each outcome occurring.**
- ***Step 1:* Simply take each numerical outcome and multiply each one by its probability.**
- ***Step 2:* Then add up all those resulting products.**
- **Note: Each outcome is *weighted* by the likelihood of that outcome occurring.**

Calculating Expected Values (continued)

- **Recall Example 1: The possible outcomes are the different possible “winnings” on the raffle ticket purchase.**
- **So the outcomes are 200, 50, or 0.**
- **The corresponding probabilities for these outcomes are: 1 / 1000, 1 / 1000, and 998 / 1000.**
- **So the *expected winnings* is: 200 $\times$ 0.001 + 50 $\times$ 0.001 + 0 $\times$ 0.998 = 0.25.**
- **Therefore a purchaser of one ticket has an “expected winnings” of $0.25, or 25 cents.**
- **Clearly the $1 price is “unfair” from the buyer’s perspective . . . but it’s for a good cause!**

Interpreting Expected Values (continued)

- **Note that the "expected value" may not in fact be one of the possible values of the variable.**
- **For the variable "raffle winnings," the expected value was $0.25, but that wasn't one of the possible values that the buyer could win.**
- **We can interpret the expected value as a *long-run average*.**
- **If the experiment were repeated many times, the *average value* of the variable across those repetitions would be near the expected value.**
- **If the buyer bought many tickets, she would win about $0.25 for each ticket purchased, *on average*.**

Clicker Quiz 1

In college football, after scoring a touchdown, a team may try for 1 point by kicking the ball through the goal posts. An unsuccessful kick attempt results in 0 points. Suppose teams are successful in such kicks 96% of the time. What is the expected point total from this type of kick attempt?

A. 1

B. 0

C. 0.96

D. 1.96

Clicker Quiz 2

In college football, after scoring a touchdown, a team may try for 2 points by running or passing the ball for a score. An unsuccessful run or pass attempt results in 0 points. Suppose teams are successful in these types of attempts 44% of the time. What is the expected point total from this type of attempt?

A. 0.44

B. 0.88

C. 2

D. 1

Another Expected Value Example

***Example 2*: A probability model for the number of vehicles owned in American households is the following (note that a negligible proportion have more than 5 vehicles):**

Number of vehicles	**0**	**1**	**2**	**3**	**4**	**5**
Proportion	**0.10**	**0.34**	**0.39**	**0.13**	**0.03**	**0.01**

The expected number of vehicles in a randomly selected American household is:

$0 \times 0.10 + 1 \times 0.34 + 2 \times 0.39 + 3 \times 0.13 + 4 \times 0.03 + 5 \times 0.01 = 1.68$ cars.

The Law of Large Numbers in the Real World

- **The *law of large numbers* says that if a random phenomenon is repeated many times, the *sample mean* of these many outcomes will be close to the expected value of the phenomenon.**

- **This assurance guarantees that lotteries will make money by setting up the prize system so that the expected winnings for a ticket buyer is less than the price of the ticket.**

- **Casinos structure their games so that the expected profit of a gambler is a bit less than zero.**

- **When lots of gamblers play, some will win money . . . but in the long run, the casino knows it will come out ahead.**

The LLN in the Real World (continued)

- **Life insurance companies set up policies knowing the probability of having to pay a claim for a given customer – they set the price of the premium so that the company's expected profit is positive.**
- ***Interesting question:*** **If the company has a positive expected profit from an insurance policy, the customer's expected profit must be negative.**
- **Does it ever make sense for a customer to buy an insurance policy?**
- ***Think*: Does the law of large numbers apply to the customer in the same way as it does to the company?**
- ***Another example*: "Deal or No Deal" game show**

Beating the Odds?

- **Some gamblers believe they have a “system” that will allow them to earn a profit while gambling.**
- **In pure games of chance, this won’t work in the long run.**
- **For any game with an expected profit that is less than zero, the gambler will lose money in the long run.**

Finding Expected Values with Simulation

- **For simple probability models, we can find the expected value using simple math.**
- **With complicated models, the calculations can become very difficult.**
- **It's often easier to estimate the expected value through simulation.**
- **We simply simulate the random phenomenon many times on a computer and keep track of the numerical outcome each time.**
- **The law of large numbers tells us that the *average* of these many outcomes will be very close to the true expected value (see example applet).**