Confidence Intervals for a Population Mean

- **Example 1: What is the mean resting pulse rate for adults who do not exercise?**
- **Again, we are interested in a *parameter*: the mean of a population of interest.**
- **To estimate this parameter, we take a *random sample* from the population.**

Confidence Intervals for a Population Mean (continued)

- **For a sample of 31 nonexercising adults, the sample mean resting heart rate was 75.0 beats per minute, with a sample standard deviation of 9.0 bpm.**
- **We know this value 75.0 is an *estimate* of the true mean of the *entire population* of adults who do not exercise.**
- **How can we make that estimate more meaningful? Use a *confidence interval*.**

Sampling Distribution of $\bar{x}$

- **Imagine taking *many repeated* samples (each of size 31) and calculating the sample mean (denoted $\bar{x}$) resting heart rate each time.**
- **The sampling distribution of $\bar{x}$ describes the pattern of those many sample mean values.**
- **The *Central Limit Theorem* tells us that when the sample size n is reasonably large, the sampling distribution of $\bar{x}$ is *approximately normal*. (See example picture)**
- **Recall: This is the *same shape* as the sampling distribution for a sample proportion.**

Sampling Distribution of $\bar{x}$ (Continued)

- **The sampling distribution of $\bar{x}$ has a mean of μ, the true population mean.**
- **The sampling distribution of $\bar{x}$ has a standard deviation of $\sigma/\sqrt{n}$.**
- **Note that the sampling distribution becomes *less spread out* when the sample size is large! (see picture)**
- **We don't know the true population standard deviation σ, so when we have a large sample, we can use the sample standard deviation s in its place.**
- **We can again use the 68-95-99.7 Rule to get an approximate 95% confidence interval for the population mean.**

Confidence Interval Formula: Population Mean

- **An approximate 95% confidence interval for a population mean may be obtained using the formula:**

$$\left(\bar{x} - 2 \times \frac{s}{\sqrt{n}}, \bar{x} + 2 \times \frac{s}{\sqrt{n}}\right)$$

- **This interval is valid if the sample size is reasonably large.**

- **Usually it works pretty well if there are at least 30 observations in the sample.**

- **Note: If the data set contains *severe outliers*, we may need many more observations (at least 100?) for this formula to be valid.**

Clicker Quiz 1

A 95% confidence interval for μ is $\left(\bar{x} - 2 \times \frac{s}{\sqrt{n}}, \bar{x} + 2 \times \frac{s}{\sqrt{n}}\right)$. For the pulse rate example ($\bar{x}$ = 75.0, s = 9.0, n = 31), what is the *margin of error* of this interval?

A. **75.0**

B. $\frac{9}{\sqrt{31}}$ **= 1.62**

C. $2 \times \frac{9}{31}$ **= 0.58**

D. $2 \times \frac{9}{\sqrt{31}}$ **= 3.23**

Clicker Quiz 2

For the pulse rate example (n = 31), suppose the sample size had been 200. How would the margin of error of the 95% interval change (assuming no change in s)?

A. It would become smaller.

B. It would become larger.

C. It would remain the same.

D. It is impossible to tell.

Confidence Interval: Pulse Rate Example

- **Recall our pulse rate example: From our sample of n = 31 adults, we found that a sample mean of $\bar{x}$ = 75.0 and s = 9.0.**
- **The left endpoint of our 95% confidence interval would thus be $75 - 2 \times \frac{9}{\sqrt{31}}$ = 75.0 - 3.23 = 71.77.**
- **The right endpoint of our 95% confidence interval would thus be $75 + 2 \times \frac{9}{\sqrt{31}}$ = 75.0 + 3.23 = 78.23.**
- **So the 95% confidence interval for the true mean resting heart rate for all nonexercising adults is (71.77, 78.23).**

Interpreting the Confidence Interval: Pulse Rate Example

- **We are 95% confident that the mean resting heart rate for all nonexercising adults is between 71.77 and 78.23 beats per minute.**
- **Again: This interval was obtained using a CI method that will “capture” the true parameter 95% of the time (i.e., in 95% of samples).**
- **If we wanted a higher or lower confidence level (99%? 90%?), then we could use the same formula, except a different number than 2 – see Table 21.1.**

Testing for Significance

- **Suppose we are interested in whether one particular value of the parameter is reasonable.**
- **Maybe this value comes from common belief, previous studies, or biological expectations.**
- ***Example:* Is the mean resting heart rate of nonexercisers significantly different from 72?**
- **If the number 72 is contained in the 95% confidence interval for μ, then we say the mean resting heart rate of nonexercisers is *not significantly different from* 72, at a 5% significance level.**
- **The *significance level* (as a percentage) is always 100% minus the corresponding confidence level.**

Clicker Quiz 3

We found a 95% confidence interval for the true proportion of adults who gamble was (0.1466, 0.1934). Is the proportion of gambling adults significantly different from 0.2, using a 5% significance level?

A. The proportion of adults who gamble is NOT significantly different from 0.2.

B. The proportion of adults who gamble is significantly different from 0.2.

C. We cannot judge the significance of proportions.

D. We cannot use a 5% significance level.