Example 1: The national average score for a certain standardized test is known to be 500. A school district has a random sample of 80 students take the test. Their scores have $\overline{Y} = 521.2$ and $s = 97.4$. Is the population mean score for the district significantly higher than the national average? Use $\alpha = .05$.

Example 2: A political candidate earned 56% of the vote in the previous election. He suspects his support is less than that this year. A random sample of 274 voters finds that 150 of those support him. Is there evidence that his support this year is less than 56%? Use $\alpha = .01$.

- Recall that in Sec. 10.2 notes, Example 2, we tested a hypothesis about p using the binomial distribution.
- This binomial test is called an <u>exact</u> test, while the z-test in Sec. 10.3 is an approximate test.
- When the sample size is large, the tests will perform similarly, but with a small sample, the <u>exact</u> test is more appropriate.

<u>Example 3</u>: A university proposes to put a building on land currently used as a surface parking lot. Of interest is whether faculty and students feel differently about the proposal. A random sample of 120 faculty found 35 supporting the proposal, while a sample of 352 students found 72 supporting the proposal. Use $\alpha = .05$.

10.4 Finding Type II Error Probabilities

- While we set $\alpha = P[\text{Type I error}]$ to be a small number, we would also like $\beta = P[\text{Type II error}]$ to be fairly small.
- For z-tests, we can calculate β for a specified value of the true parameter.

Example 1 again: Recall we test $H_0: \mu = 500$ vs. $H_a: \mu > 500$ (at $\alpha = .05$) using a sample of size 80. What is β if the true μ is 515?

- <u>Note</u>: The farther the true μ is away from μ_0 and into the alternative region, the more likely we will reject H_0 and the smaller β will be.

<u>Exercise</u>: In Example 1, if the true μ is 530, show that $\beta = 0.1335$.

<u>Example 2 again</u>: Recall we test $H_0: p = 0.56$ vs. $H_a: p < 0.56$ (at $\alpha = .01$) using a sample of 274. What is β if the true $p = 0.52$?

Note: β can be calculated similarly for two-tailed z-tests, but the calculations are slightly longer since the RR consists of the union of two mutually exclusive events.

- The only way to decrease β (while still keeping α low) is to:

- Consider an upper-tail test of $H_0: \mu = \mu_0$ vs. $H_a: \mu > \mu_0$. Suppose we specify α and our desired β when $\mu = \mu_a$ ($> \mu_0$). What sample size will we need?

Note: RR will have the form:

- This is the sample size we need to achieve the desired α and β.
- Since we must determine n before collecting our data, we will have to know (or provide a guess for) σ.

- Example 1 again: We test $H_0: \mu = 500$ vs. $H_a: \mu > 500$. We desire $\alpha = .05$, and $\beta = 0.10$ when $\mu = 515$. What sample size should we use? We guess $\sigma = 100$.

Note: The sample size formula is the same for a lower-tail z-test, assuming $\mu_a < \mu_0$.

10.6 P-values

- The α value chosen will determine the RR, which along with the data determines whether H_0 is rejected.
- But what if a reader/consumer demands a different (e.g., more stringent) protection against making a Type I error? (Perhaps ______ rather than ______)
- One recommendation is for the researcher to report the P-value of the test.

- Given the observed data and test statistic, the p-value is the smallest significance level that would lead to rejection of H_0.
- It is found by calculating the probability of observing a test statistic __________ or ______________ as the value we did observe, ______________.
- Then readers may make their own conclusions about H_0, based on their own α:
  * If the p-value is greater than my α, I would __________ H_0.
  * If the p-value is less than or equal to my α, I would __________ H_0.

Finding P-values

Example 1 again: $H_0: \mu = 500$ vs. $H_a: \mu > 500$
and the test statistic was:
- A test statistic that is extremely ________ favors H_a, so the P-value is:

- A reader with an α of .03 would ________ H_0 based on these data.
- A reader having an α of .01 would ________ H_0 based on these data.

Example 2 again: $H_0: p = 0.56$ vs. $H_a: p < 0.56$

Test statistic was:

- A test statistic that is extremely ________ favors H_a, so the P-value is:

Example 3 again: $H_0: p_1 = p_2$ vs. $H_a: p_1 \neq p_2$

Test statistic was:

- A test statistic that is ________________ ________ favors H_a, so the P-value is:

⇒ Using $\alpha = .05$, we would ______________, but using $\alpha = .04$, we would ______________ based on these data.

- P-values can also be calculated when the test statistic has a <u>discrete</u> distribution.

<u>Example 2, Sec. 10.2 notes:</u> (Defective rate example)

- We test $H_0: p = 0.03$ vs. $H_a: p > 0.03$, but suppose our sample was only 10 parts, and suppose we observed $Y = 2$ defective parts.
- Under H_0, our test statistic Y has a

- A test statistic that is extremely ________ favors H_a here:

- A consumer having an α of .05 would ____________ and conclude ____________ based on these data.

- <u>Note</u>: With test statistics having null distributions that are t, χ^2, or F, our tables only allow us to specify a <u>range</u> for the P-value, but computers and calculators typically report the precise P-value for common hypothesis tests.