<u>Example 1</u>: Let Y be a single observation from an expon(β) distribution. Find the MP test of $H_0: \beta = 1$ vs. $H_a: \beta = 2$ having $\alpha = .05$.

<u>Example 2:</u> Let $Y_1, \ldots, Y_5 \overset{iid}{\sim} \text{Pois}(\lambda)$. Find the MP test of $H_0: \lambda = 1$ vs. $H_a: \lambda = 3$ having α as close as possible to 0.05.

Theorem: Let $Y_1, \ldots, Y_n$ be iid from pdf $f(y;\theta)$ and let U be a sufficient statistic. Then the RR of the MP test of $H_0: \theta = \theta_0$ vs. $H_a: \theta = \theta_a$ depends on U.

Proof:

Uniformly Most Powerful Tests

- Suppose we perform the simple-vs.-composite test of

(assuming the distribution is completely specified except for θ).

Defn: A test is uniformly most powerful (UMP) if the RR of the MP test of $H_0: \theta = \theta_0$ vs. $H_a: \theta = \theta_a$ (where $\theta_a > \theta_0$) only depends on θ_0 (not θ_a), and thus the test is MP for any choice of θ_a ($> \theta_0$).

- It may be similarly possible to find a UMP test of $H_0: \theta = \theta_0$ vs. $H_a: \theta < \theta_0$.

Example 1: Let Y_1, Y_2 be iid Rayleigh r.v.'s having pdf

$$f(y) = \begin{cases} \frac{2y}{\theta} e^{-y^2/\theta}, & y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find the UMP test of $H_0: \theta = 1$ vs. $H_a: \theta > 1$ having $\alpha = 0.05$.

- The MP test will have RR of the form

Example 2: If $Y_1, \ldots, Y_n \overset{iid}{\sim} N(\mu, \sigma^2)$ with unknown μ and known σ^2, then Example 10.23 of the book shows that the UMP test of $H_0: \mu = \mu_0$ vs. $H_a: \mu > \mu_0$ is simply:

- Similarly, the UMP test of $H_0: \mu = \mu_0$ vs. $H_a: \mu < \mu_0$ is

- However, for some hypotheses, a UMP test does not exist.

- In the example above, there is <u>no</u> UMP test for $H_0: \mu = \mu_0$ vs. $H_a: \mu \neq \mu_0$.

<u>Why?</u> Consider the two-tailed z-test. The upper-tail z-test will have a higher power function than it for any $\mu_a > \mu_0$ and the lower-tail z-test will have a higher power function than it for any $\mu_a < \mu_0$.

Picture:

- In this case, no test procedure has <u>uniformly</u> highest power across <u>all</u> values of $\mu \neq \mu_0$.