11.11 Properties of the Least-Squares Estimators in MLR

- All of the following are basic extensions of the results we derived in SLR:

① Unbiasedness: $E(\hat{\beta}_i) = \beta_i$ for all $i = 0, 1, 2, \ldots, k$.

② $V(\hat{\beta}_i) = c_{ii}\sigma^2$ for each $i = 0, 1, 2, \ldots, k$.

c_{ii} = i-th diagonal element of $(\underset{\sim}{X}'\underset{\sim}{X})^{-1}$ if the first element is indexed "0".

③ $\text{cov}(\hat{\beta}_i, \hat{\beta}_j) = c_{ij}\sigma^2$ for all $i, j \in \{0, 1, \ldots, k\}$.

c_{ij} = (i, j) element of $(\underset{\sim}{X}'\underset{\sim}{X})^{-1}$ where the first row and first column are indexed with "0".

④ $MSE = \dfrac{SSE}{n-k-1}$ is an unbiased estimator of σ^2, where

$SSE =$

[Note the denominator becomes ______ in SLR.]

- If the ε_i's have a normal distribution, then

(5) Each $\hat{\beta}_i$ is

(6) $\dfrac{(n-k-1)MSE}{\sigma^2} \sim$

(7) Each $\hat{\beta}_i$ is independent of MSE, for $i = 0, 1, 2, \ldots, k$.

<u>Example 1, previous section</u>: Find $V(\hat{\beta}_1)$ and $\text{cov}(\hat{\beta}_1, \hat{\beta}_2)$. Estimate these by replacing σ^2 with the MSE.

From R,

11.12 Inference in MLR

- Consider the vector of constants

- Note that the linear combination

  may be expressed concisely as

- We now study inference about such linear combinations.

Example: Let $\underset{\sim}{a} = [0, 1, 0, 0, \ldots, 0]'$. Then

$\underset{\sim}{a}' \underset{\sim}{\beta} =$

Example: Let $\underset{\sim}{a} = [1, x_1^*, x_2^*, \ldots, x_k^*]'$. Then

$\underset{\sim}{a}' \underset{\sim}{\beta} =$

- So inferences about an individual β_i or about $E(Y)$ are examples of this type of inference.

- Consider the estimator of $\underset{\sim}{a}'\underset{\sim}{\beta}$:

- Then $E(\underset{\sim}{a}'\hat{\underset{\sim}{\beta}}) =$

$V(\underset{\sim}{a}'\hat{\underset{\sim}{\beta}}) =$

- Recall that these variances and covariances involve elements of $(\underset{\sim}{X}'\underset{\sim}{X})^{-1}$. In fact, $V(\underset{\sim}{a}'\hat{\underset{\sim}{\beta}})$ may be written concisely as

- Since $\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_k$ are normally distributed, $\underset{\sim}{a}'\hat{\underset{\sim}{\beta}}$ is

- Similarly as in SLR, if we estimate $V(\underset{\sim}{a}'\hat{\underset{\sim}{\beta}})$ by replacing σ^2 with MSE,

has a ______________ with __________.

- Thus __________ and CIs about $\underset{\sim}{a}'\underset{\sim}{\beta}$ can be developed analogously to SLR.

Hypothesis Test

$100(1-\alpha)\%$ CI for $\underset{\sim}{a}'\underset{\sim}{\beta}$:

Example 2, previous section: We fit a quadratic regression to the ski-racer data. Would a linear regression be sufficient? Test H_0: vs. H_a: at $\alpha = 0.05$.

Example 1, previous section: Find a 90% CI for the expected graduation rate for colleges with median SAT score 1050 and per capita expenses \$15,000.

Interpretation:

Same example: Find and interpret a 90 % CI for the marginal effect of per capita expenses in this model.

Interpretation:

Note: Most software will give the test statistic and P-value of the two-tailed t-test of $H_0: \beta_i = 0$ for each β_i, $i = 0, 1, 2, \ldots, k$.