

13.13 The Linear Model for ANOVA

- In the regression models we have studied so far, the independent variable(s) have been numerical variables that we treat as either continuous in nature, or if discrete, having a large number of levels.
- We could also conceive of a linear model in which the independent variable is a factor with only a few levels of interest.
- Such models often arise in experimental studies in which the experimenter sets some factor at a few different levels and observes how a related response variable varies.

Example: An agricultural scientist applies four different types of fertilizer to various plots in a field and then measures the resulting yields of corn crops.

Example: We compare the mean amounts of active ingredients across five different brands of aspirin.

- A linear model with only factors (i.e., having a few categories of interest) as its independent variables is called an Analysis of Variance (ANOVA) model.
- A linear model with only a single factor is called a One-way ANOVA model.
- The primary goal of the one-way ANOVA is to compare $E(y)$ across the k levels of the factor.

- The one-way ANOVA model can be written:

where n_i is the number of observations at the i -th level of the factor;

M_i is the population mean response for an observation at the i -th level and

- To fit this as a linear model, we define $(k-1)$ dummy variables.
- One level is designated as the baseline level (R uses the first level as the baseline, SAS uses the last level as the baseline).

- The dummy variables take on values 0 or 1:
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A	43.7, 54.2, 51.5, 47.7
W	46.7, 58.2, 50.5, 45.6
J	58.6, 51.1, 49.1, 54.9
S	68.4, 57.7, 56.4

-The linear model is

Let "S" be the baseline level. Then

- Of interest is whether the mean depression score varies across the four groups.
- We can answer this by testing with a Complete vs. Reduced F-test.

- If we reject this H_0 , we conclude there is a significant difference in mean depression across groups.
- We have

- Matrix algebra will show that

and

- We see that $\hat{\beta}_0 =$
- Also, as we might expect:
 - $\hat{\beta}_1 =$
 - $\hat{\beta}_2 =$
 - $\hat{\beta}_3 =$
- Our test of $H_0: \beta_1 = \beta_2 = \beta_3 = 0$ has test statistic and P-value =
- If $\alpha = 0.10$, we could _____ and conclude the mean depression score
- To compare each pair of level means, we could use the Bonferroni simultaneous comparison method discussed in Sec 13.12 (see R code on course web page).
- STAT 515 and especially STAT 516 discuss ANOVA models in much greater detail.