The Effect of the Prior On Our Posterior Inference

- With a beta(3, 1) prior, our posterior mean was
- Suppose we had used a beta(12, 4) prior (same prior mean, but ________ prior variance =).
- In that case, the posterior mean would be

- When we have ______ prior certainty, our Bayesian estimate comes closer to the prior mean than when we have ______ prior certainty.
- What if we used a beta (3, 2) prior (having prior mean)?
- Our Bayesian estimate would be

Prior	Prior Mean	MLE	Posterior Mean

- As the prior mean shifts, the posterior mean shifts with it somewhat.
- Note: With large samples, the sample estimate dominates and the effect of the prior is relatively minor.

Fact: The Bayes estimator of any function $\tau(\theta)$ of θ can be found by the posterior expected value of $\tau(\theta)$:

Example 1 again: Find the Bayesian estimator of $V(Y_i) = p(1-p)$.

<u>Example 2:</u> Let $Y_1, \ldots, Y_n$ be iid r.v.'s with pdf

$$f(y) = \begin{cases} \theta e^{-\theta y} & \text{if } y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

where $\theta > 0$. Note this is ________________ distribution with mean

- Consider a prior distribution for θ

Then

- We recognize this as the kernel of a density.
- So $g^*(\theta | y_1, \dots, y_n)$ is
- The Bayesian estimator of θ is
- The form of this makes sense since θ is

Theorem: If U is a sufficient statistic for θ, then a Bayes estimator of θ is a function of U.

Proof:

- Therefore we can find the posterior for θ using the conditional distribution of U given θ.

<u>Example 3:</u> $Y_1, \dots, Y_n \overset{iid}{\sim} N(\mu, \sigma^2)$ where μ is unknown and σ^2 is <u>known</u>. Consider a $N(\eta, \delta^2)$ prior for μ. Find the posterior distribution for μ.

- This is the kernel of a ________ density with mean

and variance

- So the posterior for μ must have this distribution.

- We see the Bayes estimator is the posterior mean

which is a weighted average of the

______________ and the ______________.

<u>Note</u>: The Bayes estimator here is

__________ (recall that

- However, it is _____________ since the bias $\to 0$ and its variance $\to 0$ as $n \to \infty$.
- Many Bayesian estimators are ________ but ______________.

Application: An Army recruiting staff gives intelligence tests to its recruits, and the scores are known to be normally distributed with variance $15^2 = 225$. The staff will estimate the mean score for next year's set of recruits using the scores of a small sample of next year's group. The staff believes the mean score will be around 100, and is 95% sure the mean score will be between 90 and 110.

- Based on the ________________, the prior for μ will have standard deviation

$\Rightarrow$

- The sample of 30 recruits yields scores with $\sum Y_i = 2850$. So

- The posterior for μ is ________________

with mean

and variance

- We see the Bayes estimate (posterior mean) of is between the prior mean of and the sample mean of

- Also, the posterior variance is ______ than the prior variance, so we have ______ certainty about μ after examining the sample.

Noninformative Priors

- Sometimes we can use the Bayesian approach even when we have no subjective prior knowledge about the target parameter.
  - We may use a <u>noninformative</u> (objective) prior.
- In Example 1, what is a natural noninformative prior about p?

- In Example 3 (the normal model), a noninformative prior for μ is

which is not even a proper density!

- It is OK if the prior is <u>improper</u> as long as the resulting posterior is a proper pdf.

<u>Note:</u>

- This is a ____________________, so the posterior for μ is

- In this case, the Bayesian posterior inference will mirror the classical inference about μ based on $\bar{Y}$.