STAT 515 Test 1 Example Solutions

#1 [Version A] / #2 [Version B]: The set of possible values has gaps, or is "countable".

#2 [V.A] / #6 [V.B]. Let A = "win", B = "sellout".

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.6 - 0.54 = \boxed{0.76}$

(b) $1 - P(A \cup B) = 1 - 0.76 = \boxed{0.24}$

(c) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.54}{0.6} = \boxed{0.9}$

(d) $P(A|B) \neq P(A)$, so A and B are NOT independent.

(e) [V.A]: $(30)(0.7) + (-50)(0.3) = \$6$

[V.B]: $(20)(0.7) + (-40)(0.3) = \$2$

#3 [v. A] / (a) $P(A \cap B) \underset{\text{indep}}{=} P(A)P(B) = (0.2)(0.5) = \boxed{0.1}$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.5 - 0.1 = \boxed{0.6}$

(c) $P(A|B) = P(A) = 0.2$ since A, B are independent

#4 [v. B]: (a) $P(A \cap B) \underset{\text{indep}}{=} P(A)P(B) = (0.4)(0.5) = 0.2$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.2 = \boxed{0.7}$

(c) $P(A|B) = P(A) = 0.4$ since A, B are independent.

#4 [vA] / #3 [vB]: (a) bar graph, categorical.

(b) 35 (c) $\frac{10}{35} \approx .28$

#5 [vA] / #5 [vB]: (a) box plots, numerical

(b) They have a similar spread.
Drug 2 has a distribution with a greater center.

(c) Drug 1: Median ≈ 0.3 hours
Drug 2: Median ≈ 1.9 hours

#6 [v. A] / #9 [v. B]: $X \sim \text{binomial}(n=6,\ p=0.80)$

(a) $P(X \geq 4) = 1 - P(X \leq 3) = 1 - .099 = .901$

(b) $P(X \leq 2) = .017$

(c) $P(X=3) = P(X \leq 3) - P(X \leq 2) = .099 - .017 = .082$

(d) $\mu = np = (6)(.80) = 4.8$

#7 [v. A] (a) all $P(x) \geq 0$, $\sum P(x) = .45 + .40 + .15 = 1$ ✓

(b) $\sum x P(x) = (0)(0.45) + (1)(0.40) + (2)(0.15) = 0.7$

(c) $\sum x^2 P(x) = (0^2)(0.45) + (1^2)(0.40) + (2^2)(0.15) = 1.0$

$\sigma^2 = 1.0 - (0.7)^2 = 0.51 \Rightarrow \sigma = \sqrt{.51} = 0.714$

#1 [v. B] (a) all $P(x) \geq 0$, $\sum P(x) = .45 + .35 + .20 = 1$ ✓

(b) $\sum x P(x) = (0)(0.45) + (1)(0.35) + (2)(.20) = 0.75$

(c) $\sum x^2 P(x) = (0^2)(0.45) + (1^2)(0.35) + (2^2)(0.20) = 1.15$

$\sigma^2 = 1.15 - (0.75)^2 = 0.5875 \Rightarrow \sigma = \sqrt{.5875} = 0.7665$

#8 [v A] Let A = paperback, B = Boston

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)} = \frac{(.25)(.60)}{(.25)(.60) + (.35)(.40)} = .5172$$

#8 [v B]

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)} = \frac{(.25)(.60)}{(.25)(.60) + (.45)(.40)} = .4545$$

#9 [v. A] $\lambda = 8.5 \Rightarrow$ (a) $P(5) = \dfrac{e^{-8.5}(8.5)^5}{5!} = 0.075$

(b) Let X = morning cars, Y = evening cars

$E(X+Y) = E(X) + E(Y) = 8.5 + 6.5 = 15$

#7 [v. B] $\lambda = 6.5 \Rightarrow$ (a) $P(5) = \dfrac{e^{-6.5}(6.5)^5}{5!} = 0.1454$

(b) $E(X+Y) = E(X) + E(Y) = 6.5 + 4.5 = 11$