STAT 515 --- STATISTICAL METHODS

Statistics: The science of using data to make decisions and draw conclusions

Two branches:

Descriptive Statistics:

- **The collection and presentation (through graphical and numerical methods) of data**
- **Tries to look for patterns in data, summarize information**

Inferential Statistics:

- **Drawing conclusions about a large set of individuals based on data gathered on a smaller set**
- **Example:** Relationship between high school GPA and college GPA. Gather a "few" college students and observe/measure their HS + college GPAs. - Make conclusions about "all" college students.

Some Definitions

Population: The complete collection of units (individuals or objects) of interest in a study

Variable: A characteristic of an individual that we can measure or observe

Examples: GPAs of students
Life expectancy of Americans
Races of professors
Preferences of consumers

Sample: A (smaller) subset of individuals chosen from the population

- **In statistical inference, we use *sample data* (the values of the variables for the sampled individuals) to make some conclusion (e.g., an estimate or prediction) about the population.**

Example: If we want to ~~measure~~ determine the average cholesterol level of residents of Columbia. Difficult or impossible to measure chol. level for all residents.
- Take a sample of residents and estimate the overall average chol. level using the sample average (mean).

How reliable is this generalization to the population? For inference to be useful, we need some genuine measure of its reliability.

Types of Data:

Quantitative (Numerical) Data: Measurements recorded on a natural numerical scale (can perform mathematical operations on data).

Qualitative (Categorical) Data: Measurements classified into one of several categories.

Examples: Sizes of families (Quantitative)
GDP of countries (Quantitative)
Races of people (Qualitative)
Colors of T-shirts (Qualitative)

Sources of Data:

- **Published Source: Many government, business, financial, and sports statistics are collected and archived in publications or online.**
- **Designed Experiment: Researcher imposes a treatment on individuals, then observes responses. (Researcher maintains strict control --- often in lab setting.)**

Example: Clinical Trials for new Drugs

- **Surveys: Researcher selects sample of individuals and records their responses to questions**

Example: Political Polls
U.S. Census Customer Service Satis. Cards

- **Observational Study: Researcher observes individuals and measures variables, but has no control over process being observed.**

Example: - Study to measure weights of animals born in ZOO.
- Traffic study
- Amount of children born to S. Carolina families

Typically, an experiment is better for establishing cause/effect between two variables, but it's not always practical or possible.

Regardless of the type of study, we must ensure that we have a representative sample, one that has similar characteristics to the population.

The best kind of sample is a *simple random sample* (every subset has an equal chance of being selected)

Standard statistical methods assume the data are a random sample from the population.

Methods for Describing Data Sets

Important Principle in Statistics: Data Reduction

Example: Study of household incomes of USC students. Take a random sample of 100 students, and write down their family ~~incomes~~ incomes.

- **List of 100 numbers would be confusing, not informative**
- **Need to reduce data to a reasonable summary of information.**

Two ways:

- **Graphs and Plots**
- **Numerical Statistics**

Describing Qualitative Data

- **Data are categorized into classes**
- **The number of observations (data values) in a class is the class frequency** (count)
- **Class Relative Frequency = class frequency / *n*** ← total number of data values
- **The CRF's of all classes add up to 1**

Example:

Table lists three types of aphasia (impairment of language ability) for a sample of 22 people:

Class	Frequency	CRF
Broca's	5	$\frac{5}{22} = .227$
Conduction	7	$\frac{7}{22} = .318$
Anomic	10	$\frac{10}{22} = .455$

Graphical Displays:

Bar graph: Height of bars indicates frequencies for each category (see p. 30) 32

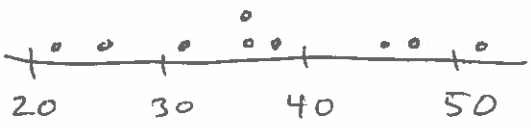
Pie chart: Area of "pie slices" indicates relative frequency for each class (see p. 30) 32

Describing Quantitative Data

To detect and summarize patterns in a set of numerical data, we use:

- **Dot Plots: these represent each data value with a dot along a numerical scale. When data values repeat, the dots pile up vertically at that value.**
- **Stem-and-leaf Display (good for small data sets): Separate each number in a data set into a stem and a leaf (usually the last digit). There is a column of all the stems in the data set, and at each stem, the corresponding leaf digits line up to the right.**

Example: Data: 22, 26, 31, 36, 36, 38, 44, 46, 51



2	2 6
3	1 6 6 8
4	4 6
5	1

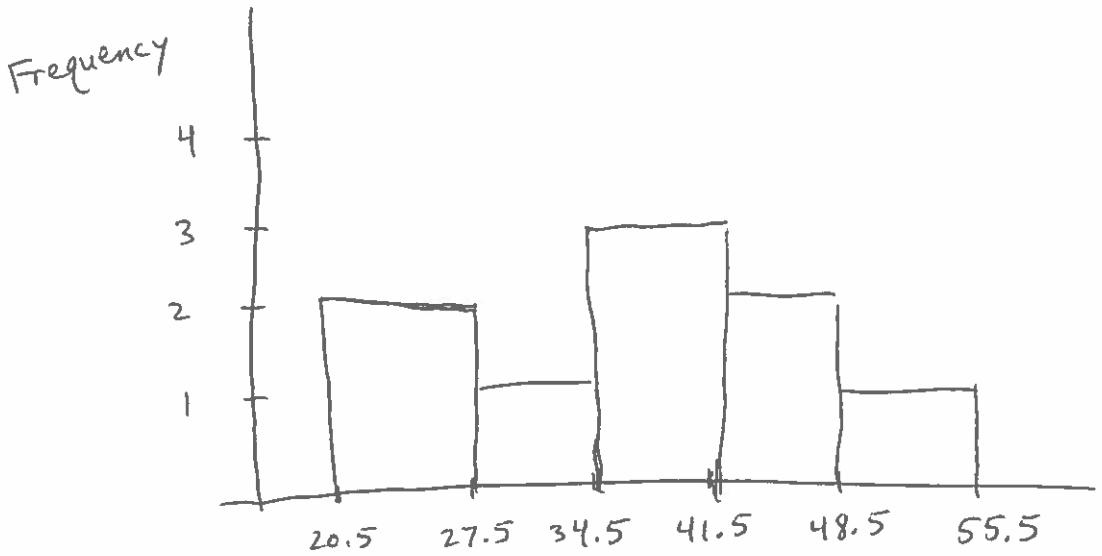
See p. 39 for another example. 44

- **Histogram: Numerical data values are grouped into measurement classes, defined by equal-length intervals along the numerical scale.**

Like a bar graph, a histogram is a plot with the measurement classes on the horizontal axis and the class frequencies (or relative frequencies) on the vertical axis.

For each measurement class, the height of the bar gives the frequency (or RF) of that class in the data.

Example: 22, 26, 31, 36, 36, 38, 44, 46, 51



Guidelines for Selecting Measurement Class Intervals:

- **Use intervals of equal width**
- **Each data value must belong to exactly one class**
- **Commonly, between 5 and 12 classes are used**

Note: Different choices of Class Intervals (in position and number) may produce different-looking histograms.

Most often, we use software to help choose intervals and create histograms.

Histograms don't show individual measurement values (stem-and-leaf displays and dot plots do).

But for large data sets, histograms give a cleaner, simpler picture of the data.

Summation Notation

In statistics, we customarily denote our data values as $x_1, x_2, \ldots, x_n$. (n is the total number of observations.)

The sum of a set of numbers is denoted with Σ.

That is, $x_1 + x_2 + \ldots + x_n = \sum_{i=1}^{n} x_i = \sum x_i$

Or, we might sum the squared observations:

$x_1^2 + x_2^2 + \ldots + x_n^2 = \sum_{i=1}^{n} x_i^2$

Example:

If our data are: 1, 2, 8, 5 $(n = 4)$

then $\sum x_i = 1 + 2 + 8 + 5 = 16$

and $\sum x_i^2 = 1 + 4 + 64 + 25 = 94$