

STAT 520 – Homework 3A – Fall 2025

1) Consider the MA(2) process, where all the $\{e_t\}$ values are independent white noise with variance σ_e^2 :
$$Y_t = e_t - 0.5 e_{t-1} - 0.3 e_{t-2}$$

- Find $\text{cov}(Y_t, Y_t) = \text{var}(Y_t)$.
- Find $\text{cov}(Y_t, Y_{t-1})$ and, from this, find the lag-1 autocorrelation $\text{corr}(Y_t, Y_{t-1})$.
- Find $\text{cov}(Y_t, Y_{t-2})$ and, from this, find the lag-2 autocorrelation $\text{corr}(Y_t, Y_{t-2})$.
- Argue that $\text{cov}(Y_t, Y_{t-k}) = 0$ for all $k \geq 3$.

2) **[Mandatory for graduate students, extra credit for undergraduates]**

Repeat all parts of Problem 1, but for the process:

$$Y_t = e_t - 1.2 e_{t-1} + 0.7 e_{t-2}$$

3) Consider the AR(1) process: $Y_t = \phi Y_{t-1} + e_t$, where e_t is a white noise process with mean zero and variance $\sigma_e^2 > 0$.

Show that if $|\phi| = 1$, the process cannot be stationary. [Hint: Take variances of both sides.]

4) The monthly U.S. air passenger miles flown for January 1960 to December 1971 are in the `airpass` object in the `TSA` package. Type `library(TSA); data(airpass); print(airpass)` in R to see the data set.

- Plot the time series, using plotting symbols that allow you to check for seasonality. What basic pattern do you see from the plot?
- Plot the (natural) log-transformed time series. What basic pattern do you see from the plot? What effect has the log transformation had?
- Plot the differences of the natural logarithms. Does this plot suggest that a stationary model might be appropriate for the differences of the natural logarithms? Briefly explain.
- Plot the fractional relative differences, $(Y_t - Y_{t-1})/Y_{t-1}$, which can be obtained in R with the code:
`diff(airpass) / (zlag(airpass)[-1])`

How do these values compare with the differences of the natural logarithms from part (c)?