

STAT 520 – Homework 6 – Fall 2025

Note: Problems 1, 2, and 3 are mandatory for everyone. Problems 4 and 5 are mandatory for graduate students and extra credit for undergraduates.

1) The quarterly earnings per share for 1960-1980 are in the `JJ` object in the `TSA` package. Type `library(TSA); data(JJ); print(JJ)` in R to see the data set.

- (a) Plot the original time series and the (natural) logarithm of the series. Explain why the log transformation is useful when modeling this series.
- (b) Explain why the log-transformed series is clearly not stationary. Plot the first differences of the log-transformed series. Could the differenced logged series be viewed as stationary?
- (c) Graph the sample ACF of the differenced logged data. What does this plot suggest?
- (d) Take both the first differences and the (lag-4) seasonal differences of the logged data. Plot this differenced and seasonally differenced series. Interpret what the plot indicates.
- (e) Plot the sample ACF of the differenced and seasonally differenced logged series. Interpret what the plot tells you.
- (f) Fit an $\text{ARIMA}(0,1,1) \times (0,1,1)_4$ model to the logged series, and assess the significance of the estimated coefficients.
- (g) Perform model diagnostics, based on the residuals.
- (h) Calculate the forecasts for the next 8 quarters of the series. Plot the forecasts along with 95% prediction intervals.

2) A data set of public transportation boardings in Denver from August 2000 through March 2006 are in the `boardings` object in the `TSA` package. These data are *already* logged: You can type `library(TSA); data(boardings); log.boardings = boardings[,1]; print(log.boardings)` in R to see the data set.

- (a) Give a time series plot of these data. Include plotting symbols for the months that help you assess seasonality. Comment on the plot and any seasonality. Is it reasonable to use a stationary model for this time series?
- (b) Plot the sample ACF of the series. Interpret what the plot tells you.
- (c) Fit an $\text{ARMA}(0,3) \times (1,0)_{12}$ model to the data, and assess the significance of the estimated coefficients.
- (d) Overfit with an $\text{ARMA}(0,4) \times (1,0)_{12}$ model to the data, and interpret the results.

3) A data set of weekly sales and prices of Bluebird Lite potato chips are in the `bluebirdlite` object in the `TSA` package. (This is for a “lite” version of the potato chips, so the data are somewhat different from the example we did in class on the `bluebird` data set.) The sales are *already* logged. Type `library(TSA); data(bluebirdlite); print(bluebirdlite)` in R to see the data set. And type `data(bluebirdlite); log.sales=bluebirdlite[,1]; price=bluebirdlite[,2]` to access the individual time series.

- (a) Use the `prewhiten` function to obtain the cross-correlation function of the prewhitened version of logged sales and price. What can be discerned about the cross-correlations at the various lags?
- (b) Fit an ordinary least squares (OLS) regression of log sales against price.
- (c) Plot the ACF and PACF of the residuals from the OLS fit in part (b). Also give the EACF of the residuals from the OLS fit in part (b). What do these tools tell you about the specification of the noise process?

(d) Fit WLS regressions of log sales against price, specifying the following models for the noise process: ARMA(4,0); ARMA(1,1); and ARMA(4,1). What model is preferred, and why?
(e) Look at the model diagnostics based on the residuals from your preferred model from (d). What do the diagnostics indicate?

4) A data set of 82 measurements from a machining process are in the `deere1` object in the `TSA` package. Type `library(TSA); data(deere1); print(deere1)` in R to see the data set.

(a) Fit an AR(2) model to the full data set. Plot the standardized residuals from this model, and the sample ACF of the residuals. What do these diagnostics tell you?

(b) Detect either additive outliers and/or innovative outliers from the model in (a). What is your conclusion?

(c) Fit an AR(2) model that incorporates the most notable outlier into the model. Plot the standardized residuals from this model, and the sample ACF of the residuals. What do these diagnostics tell you? Compare the fitted model in part (a) to the fitted model in part (c).

5) For the intervention effect model $m_t = \delta m_{t-1} + \omega S_{t-1}^{(T)}$, find the half-life of the intervention effect when $\delta = 0.6$. Also find the half-life of the intervention effect when $\delta = 0.9$.