Homework 1 – STAT 704

1. Let $Y_{11}, \ldots, Y_{1n_1}$ be a sample from a population with mean μ_1 and variance σ_1^2. Let $Y_{21}, \ldots, Y_{2n_2}$ be a sample from another population with mean μ_2 and variance σ_2^2. Define

$$\bar{Y}_1 = \sum_{j=1}^{n_1} \frac{Y_{1j}}{n_1}, \quad \bar{Y}_2 = \sum_{j=1}^{n_2} \frac{Y_{2j}}{n_2}.$$

   (a) Find $E(\bar{Y}_1 - \bar{Y}_2)$.

   (b) If $\bar{Y}_1$ and $\bar{Y}_2$ are independent, find $var(\bar{Y}_1 - \bar{Y}_2)$.

   (c) If the two populations are normal, then does $\bar{Y}_1 - \bar{Y}_2$ have a normal distribution? Explain why or why not.

2. Suppose $Y_1, Y_2, \ldots, Y_n$ are independent random variables with mean μ and variance σ^2.

   (a) Show that

$$(n-1)S^2 = \sum_{i=1}^{n} Y_i^2 - n\bar{Y}^2.$$

   (b) Show that $E(S^2) = \sigma^2$. (Hint: Use the fact that $var(Y) = E(Y^2) - [E(Y)]^2$.)

3. Let Y_1, Y_2, Y_3 be independent random variables with means μ_1, μ_2, μ_3 and a common variance σ^2. Define

$$\bar{Y} = \frac{1}{3} \sum_{i=1}^{3} Y_i.$$

   (a) Find the covariance between $Y_1 - \bar{Y}$ and $\bar{Y}$.

   (b) Find the expected value of $(Y_1 + 2Y_2 - Y_3)^2$.

4. Let Y_1 and Y_2 be random variables with expected values μ_1 and μ_2 and variances σ_1^2 and σ_2^2.

   (a) Show that $cov(Y_1 + Y_2, Y_1 - Y_2) = \sigma_1^2 - \sigma_2^2$.

   (b) If $W = Y_1 + Y_2$ and $V = Y_1 - Y_2$, then under what condition(s) can we be assured that W and V are independent random variables?