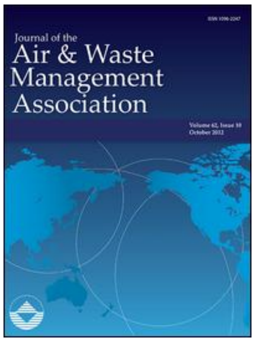

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Modeling the Energy Content of Municipal Solid Waste Using Multiple Regression Analysis

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ABSTRACT
In this research multiple regression analysis was used to develop predictive models of the energy content of municipal solid waste (MSW). The scope of work included collecting waste samples in Kaohsiung City, Taiwan, characterizing the waste, and performing a stepwise forward selection procedure for isolating variables. Two regression models were developed to correlate the energy content with variables derived from physical composition and ultimate analysis. The performance of these models for this particular waste was superior to that of equations developed by other researchers (e.g., Dulong, Steuer) for estimating energy content. Attempts at developing regression models from proximate analysis data were not successful.

INTRODUCTION
This research study was primarily motivated by the waste disposal problems of Kaohsiung City (area = 153.6 square kilometers; population = 1,488,000), the largest city in southern Taiwan. The daily production of the MSW in Kaohsiung City is 1,702 tons/day, corresponding to a generation rate of 1.14 kg/capita-day. At present the MSW is disposed of in the Shichinpu landfill, the only existing landfill in this city, located at the northern boundary of Kaohsiung City. This landfill facility is expected to be full at the end of 1995. To address the future MSW disposal needs, the Kaohsiung City government is in the process of designing three MSW incinerators.

IMPLICATIONS
An assessment of the energy content of municipal solid waste (MSW) is critical for the effective design and operation of incineration systems for waste destruction. This research establishes that individual MSW treatment facilities can use multiple regression analysis to successfully develop models for predicting the energy content of their waste from physical composition and elemental analysis. For the MSW addressed in this research, the regression models proved to be a more accurate alternative for predicting energy content than equations developed by other researchers.

One of the critical needs for MSW incineration design is to know the energy content of the waste. At present there are three types of models that are used to predict this based on one of the three following types of analysis:

- Physical composition[1]
- Ultimate analysis[2,3]
- Proximate analysis[1]

The physical composition analysis is based on the levels of plastics, paper, water, and garbage (food wastes, textiles, and garden wastes) in MSW. The ultimate analysis of waste typically involves determination of C (carbon), H (hydrogen), O (oxygen), N (nitrogen), and S (sulfur), while the proximate analysis includes an assessment of the levels of moisture, volatile combustible matter, fixed carbon, and ash.

Table 1 presents a summary of the equations/models commonly used to predict the energy content of MSW, based on the analytical profiles listed above. The focus of this research is on applying multiple regression analysis to develop predictive relationships between energy content, and the physical and chemical properties of MSW (defined by the proximate, ultimate, and physical composition analyses). The research objectives also included comparing the effectiveness of the multiple regression models developed in this study with the models listed in Table 1.

MATERIALS AND METHODS
This section presents the sampling protocol, the methods for determining the characteristics of MSW from Kaohsiung City, and the strategy for performing the multiple regression analysis.

Sampling Protocol
Each of the 11 administrative districts were sampled at random either two or three times a year. Districts with waste production of less than 300 tons/day were sampled twice, while districts with waste production of greater than 300 tons/day were sampled three times. The waste from the assigned truck was dumped onto a clean, impervious floor where it was mixed rapidly with a shovel and quartered. One shovelful was extracted from each quarter, then was

Table 1. Models/equations for predicting the energy content of municipal solid waste.[1-3]

1. Models Based on Physical Composition Analysis.

Conventional Equation

$$Hn = 88.2R + 40.5(G+P) - 6W \quad (1A)$$

Where: Hn = Net calorific value (Kcal/Kg).
R = Plastics, percent weight on dry basis.
G = Garbage, percent weight on dry basis.
P = Paper, percent weight on dry basis.
W = Water, percent weight on dry basis.

2. Models Based on Ultimate Analysis.

Dulong's equation

$$Hn = 81C + 342.5(H-O/8) + 22.5S - 6(9H+W) \quad (3A)$$

Where: C = Carbon, percent by weight.
H = Hydrogen, percent by weight.
O = Oxygen, percent by weight.
S = Sulfur, percent by weight.

Steuer's equation

$$Hn = 81(C-3xO/8) + 57x3xO/8 + 345(H-O/16) + 25S - 6(9H+W) \quad (3B)$$

Scheurer-Kestner's equation

$$Hn = 81(C-3xO/4) + 342.5H + 22.5S + 57x3xO/4 - 6(9H+W) \quad (3C)$$

3. Models Based on Proximate Analysis

Traditional Equation

$$Hn = 45B - 6W \quad (2A)$$

Where, B = Combustible volatile matter in MSW (%).

Bento's equation

$$Hn = 44.75B - 5.85W + 21.2 \quad (2B)$$

mixed and quartered again. This procedure was repeated until a refuse volume of about 0.1 m^3 was obtained (ASTM D75 and E38).

Waste Characteristics

In this research, ASTM methods[4] were used to determine the characteristics of the MSW. These methods are briefly summarized below.

- Specific weight (ASTM E1109). Refuse was placed in a 0.1 m^3 stainless steel container and weighed.
- Moisture content (ASTM E949). The weight loss was determined for duplicate freshly ground samples (50 to 100 g) dried for three to five days in a forced air oven.
- Volatile matter and ash content (ASTM E897). Three to six grams of ground and dried (105 °C) samples were combusted at 800 °C. The weight loss was termed volatile matter and the remaining mass was considered to be ash.
- Physical composition (ASTM E889). The sample was randomly subdivided and plastics, garbage, and paper were separated from two of the subsamples by hand and weighed.
- Energy content (ASTM E711). The energy content of the waste was determined by burning a weighed sample in an oxygen bomb calorimeter.
- Elemental analysis (C [ASTM E777], H [ASTM E777], O, S [ASTM E775], and N [ASTM E778]). To measure carbon and hydrogen, the sample was burned in a tube furnace and the water and CO_2 produced was absorbed and analyzed. Nitrogen concentration was measured by the Kjeldahl-Gunning method. Sulfur was measured by conversion to sulfur-dioxide, followed by absorption in hydrogen peroxide solution, and titration with barium acetate solution. The concentration of oxygen was calculated by difference.

Multiple Regression Analysis

Regression analysis was performed using SYSTAT statistical software. Attempts were made to develop three types of models. The first was based on physical composition of MSW, while the second and third models were based on the proximate and ultimate analysis, respectively. In all the models the dependent variable was the energy content. The independent variables tested for the physical composition model were the concentration of plastics, garbage, paper, and water (percent weight on dry basis); the independent variables tested for the model based on proximate analysis were combustible volatile matter, ash, and water. Finally, the parameters tested for the elemental analysis model were C, H, O, S, and N.

The stepwise forward selection procedure was used to select the best fitting regression model of the form:

$$y=\beta_0+\beta_1X_1+\beta_2X_2+\beta_3X_3+........+\beta_kX_k+E \quad (1)$$

β_o, β_1, ... β_k are the regression coefficients; and X_1, X_2, ...X_k are the selected independent variables. E is the error term representing the magnitude of y not accounted for by the other terms in the equation. The following paragraphs summarize the individual steps in this procedure.[5]

Step 1. The first variable selected to enter the model was the one most highly correlated with the dependent variable, and subsequently the associated straight-line regression equation was fitted. The most highly correlated variable would be the one with the highest R^2, which reflects the strength of the straight line relationship, with R^2 being defined as:

$$R^2 = \frac{SSY - SSE}{SSY} \quad (2)$$

where SSY = Total sum of squares (e.g., sum of squares of observed y's from mean y)
$= \Sigma\, (y_i - y_{mean})^2 =$
SSE = Error sum of squares (e.g., sum of squares of deviations of observed y's from the fitted regression line)
$= \Sigma\, (y_i - y_{predicted})^2$
The straight line regression equation relating energy content with the selected variable would be:

$$y = \beta_0 + \beta_1 X_1 + E \quad (3)$$

Next, the hypothesis that $\beta_1 = 0$ was tested by determining if the *F* statistic for the regression equation is significant by comparing it with $F_{k,n-k-1,1-\alpha}$. The *F* statistic is defined as follows:

$$F = \frac{\text{MSregression}}{\text{MSresidual}} = \frac{(\text{SSY-SSE})/k}{\text{SSE}/(n\text{-}k\text{-}1)} \quad (4)$$

In the above equation, MS regression denotes the mean sum of squares accounted by the regression equation, while MS residual represents the mean sum of squares for the residual. Note that n is the number of observations, k is the number of independent variables, and α is a preselected level of significance. If the F statistic was not significant (e.g., $F_{calculated} < F_{k,n-k-1,\, 1-\alpha}$), it was concluded that no independent variables were important predictors. If the F statistic was significant, the variable was included in the model, before proceeding to step 2.

Step 2. The partial F statistic associated with each remaining variable was calculated based on a regression equation containing that variable and the variable initially selected. The partial F statistic is defined as:

$$F(X^* | X_1, X_2, ...X_p) = \frac{SS(X^* | X_1, X_2, ...X_p)/1}{\text{MSresidual}(X_1, X_2, ...X_p, X^{**})} \quad (5)$$

$SS\,(X^* | X_1, X_2, X_3, ...X_p)$
$= \text{Regression SS}\,(X_1, X_2, X_3, ...X_p, X^*) - \text{Regression SS}\,(X_1, X_2, X_3, ...X_p)$
= Extra sum of squares explained by adding X^* to an existing regression equation having independent variables X_1 through X_p

Note: The "1" in the numerator arises from the number of terms being added, in this case one. In this case the partial F statistic would involve only two variables, and would be of the form $F\,(X_2|X_1)$.

Step 3. The variable with highest partial *F* statistic was selected.

Step 4. The significance of the partial *F* statistic associated with the variable selected in step 3 was tested.

Step 5. If this test was significant (Fcalculated $> F_{1,n-p-2,\, 1-\alpha}$), the new variable was added to the regression equation. If this test was not significant, only the variable added in step 1 was used. Note: The overall *F* statistic ($F_{k,n-k-1,\, 1-\alpha}$) is different from the partial *F* statistic defined above.

Step 6. At each subsequent step, the partial *F* statistic was determined for those variables not yet in the model, and the variable with the highest partial F value was added to the model, if it was statistically significant. At any step, if the largest partial F was not significant, no more variables were included, and the process was terminated.

Regression Diagnostics

The equations derived from the stepwise forward method enumerated above were subjected to residuals analysis. In addition, model validation was performed by plotting the measured and predicted values, and also by performing a simple regression procedure on the measured and predicted values. The model graphs show the theoretical "line of perfect fit" having a slope of 1.0 and intercept of 0, and the regression line. Perfect model simulation would be indicated by a regression line having a slope of 1.0, intercept of 0, and R^2 of 1.00, essentially overlapping the "line of perfect fit." Residuals analysis was performed by plotting the jackknife residuals against the predicted value of the dependent variable. A horizontal band of randomly distributed points would indicate that there were no systematic trends (e.g., no need to include curvilinear terms in the equation), and the assumption of a linear relationship between the dependent variables and the independent variable was valid.

RESULTS AND DISCUSSION

This section summarizes the waste characteristics, and presents three regression models developed with the protocol specified above. The section also includes an assessment of their ability to predict the energy content of MSW, and a comparison of these models with the equations developed by other researchers (see Table 1).

Characteristics of MSW from Kaohsiung

Table 2 presents the characteristics of the MSW from Kaohsiung City, including energy content, proximate and ultimate analysis, and physical composition. The data were collected during 1993. Average values and ranges for the various constituents have been included. The data suggest considerable variation in concentrations of all constituents, supporting a case for multiple regression analysis. The complete database is found elsewhere.[6]

Figure 1 shows the change in the composition (e.g., paper, plastics, garden wastes, food waste, and plastics) of Kaohsiung City MSW between 1983 and 1993. This MSW contained large amounts of food, plastics, and paper, and low concentrations of garden wastes and textiles. The ten-year period was characterized by an increase in the levels of plastics and paper in the MSW. The most probable cause for this was the increase in the number of offices, and the greater use of plastics for food packaging. Figure 2 presents the

Table 2. Characteristics of the municipal solid waste (MSW) from Kaohsiung City, Taiwan.

Constituent	*Average*	*Range*
Specific weight (kg/m³)	253	110-282
Physical Composition		
Food waste (%)	10.05	9.08-25.09
Paper (%)	37.03	17.04-43.49
Textiles (%)	6.59	0.60-20.27
Plastics (%)	15.66	10.95-30.42
Rubber and leather (%)	0.98	0.00-10.06
Garden wastes (%)	3.65	1.00-9.41
Metals (%)	6.74	2.59-17.20
Ceramics (%)	0.27	0.00-6.04
Dirt, ash, bricks, etc.	2.00	0.17-5.29
Glass (%)	7.58	0.00-17.00
Miscellaneous (%)	1.45	0.00-8.09
Ultimate Analysis		
Carbon (%)	17.85	9.00-29.72
Hydrogen (%)	2.74	1.21-3.49
Nitrogen (%)	0.51	0.29-0.74
Sulfur (%)	0.20	0.11-0.42
Oxygen (%)	11.44	6.81-18.20
Proximate Analysis		
Moisture (%)	50.27	36.7-60.49
Volatile Matter (%)	33.06	17.95-49.22
Ash (%)	16.67	9.41-27.63
Energy Content		
Gross calorific value (kcal/kg)	2154	1560-2610
Net calorific value (kcal/kg)	1704	1075-2381

changes in the gross and net calorific value of MSW between 1983 and 1993. The steady increase in the calorific value is due to increased quantities of paper and plastics, and lower levels of food wastes, which decreased the moisture content, and increased the amount of volatile matter.

Physical Composition Model

Multiple regression analysis on the physical composition data yielded the following model:

$$Hn = 2229.91 + 28.16R + 7.90P + 4.87G - 37.28W \quad (6)$$

Where:

Hn = Net calorific value (kal/kg).
R = Plastics, percent by weight.
P = Paper, percent by weight.
G = Garbage (e.g., food wastes, textiles, garden wastes), percent by weight.
W = Water, percent by weight.

The statistical parameters associated with the model are as follows:

α = 0.05
R^2 = 0.967
F = 214.55
n (number of samples) = 34

The residual analysis confirmed that the assumption of a linear relationship was valid, and there was no systematic error. As expected, the model indicates that plastics, paper, and garbage contribute positively to the energy content, while water is negatively correlated. The stepwise forward technique for regression analysis revealed water to be the most important variable, followed by plastics, paper, and garbage. Figure 3, which compares the measured and predicted energy content, represents an attempt to validate the results, and also to

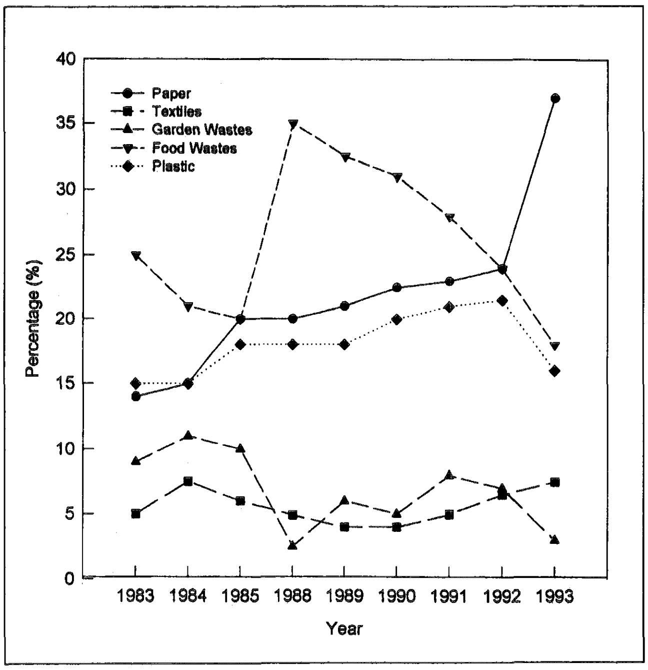

Figure 1. Compositional changes in Kaohsiung City MSW between 1983 and 1993.

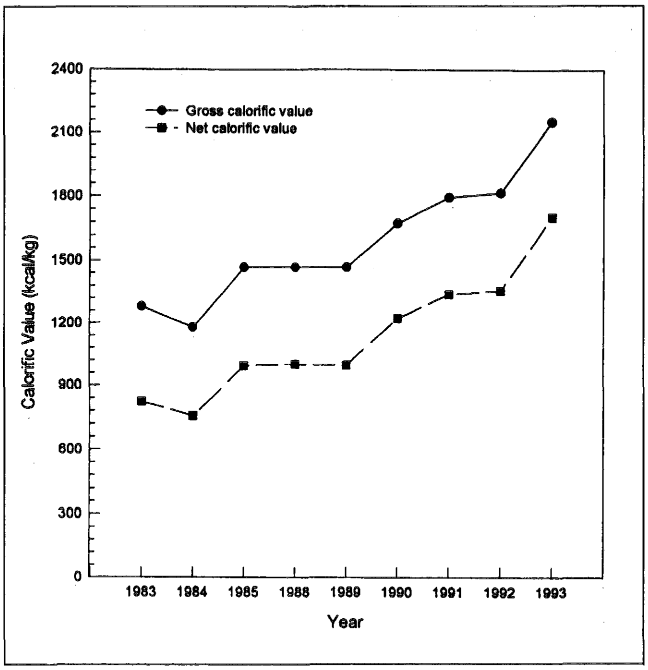

Figure 2. Change in the calorific value of Kaohsiung City MSW between 1983 and 1993.

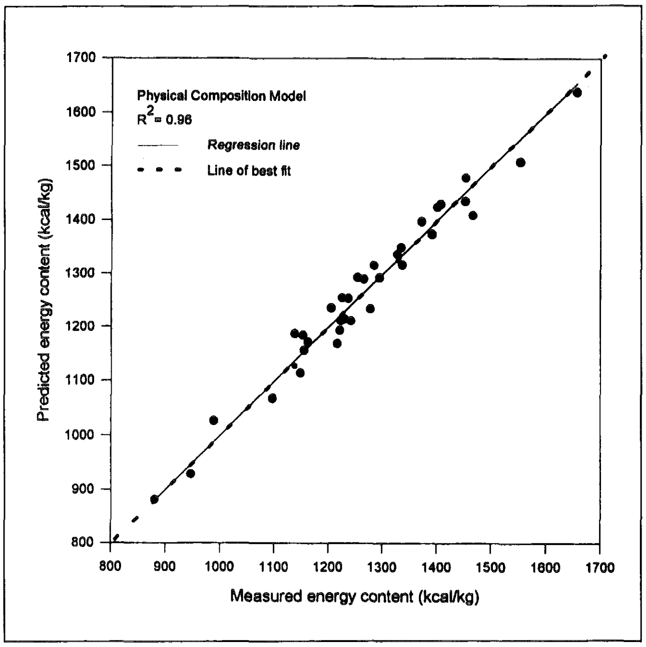

Figure 3. Comparison of measured energy content and the predicted values obtained from physical composition model developed by multiple regression analysis.

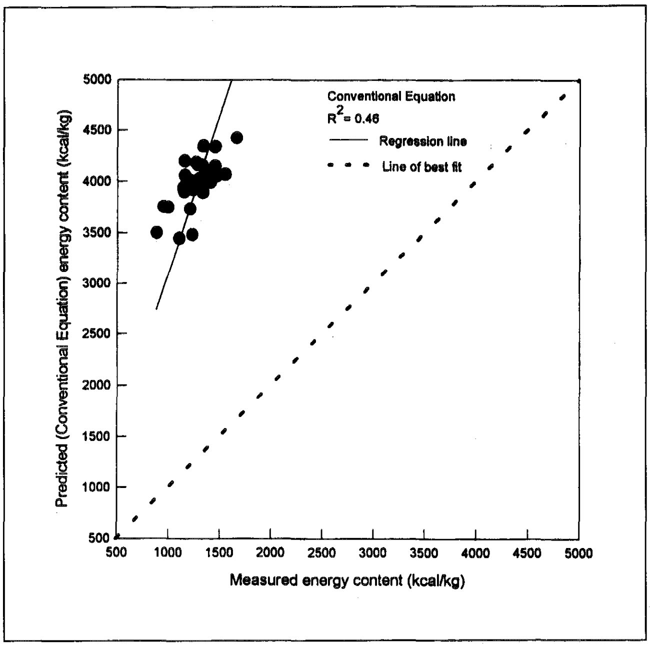

Figure 4. Comparison of measured energy content and the predicted values obtained from the conventional equation (see Table 1).

assess the accuracy of the model. The measured and predicted values agree well ($R^2 = 0.96$) over the complete range of energy contents. Most notably, the regression line is virtually coincident with the "line of best fit."

A comparison of the accuracy of this multiple regression model with the conventional equation for estimating energy content based on physical composition (see equation 1A in Table 1) is shown in Figure 4. The conventional equation substantially overpredicts the energy content of the MSW. In addition, the coefficient of determination (R^2) was only 0.46. Thus the multiple regression model developed in this work is a vastly superior tool for predicting the energy content of MSW of Kaohsiung City, compared to the conventional equation.

Ultimate Analysis Models

The regression model developed using ultimate analysis parameters (e.g., C, H, O, N, S, water) as possible independent variables is as follows:

$$Hn = 1558.80 + 19.96C + 44.30O - 671.82S - 19.92W \quad (7)$$

Where:

C = Carbon, percent by weight.
O = Oxygen, percent by weight.
S = Sulfur, percent by weight.
W = Water, percent by weight

The statistical parameters associated with the model are as follows:

α = 0.05
R^2 = 0.926
F = 109.84
n (number of samples) = 40

The model suggests that the energy content is a function of the carbon, oxygen, sulfur and water of the MSW. It is negatively correlated with moisture content and sulfur, while it is positively correlated with the remaining two elements. Regression analysis revealed oxygen to be the most important variable, followed by water, sulfur, and carbon. An interesting finding is that hydrogen did not contribute statistically to the energy content. This finding is in contrast to the results of Dulong, Steuer, and Scheurer-Kestner (see Table 1), who found that hydrogen was one of the parameters along with carbon, oxygen, and sulfur which contributed positively to the energy content.

Sulfur results in a negative contribution to the model developed in this work, possibly because the dissociation energy for sulfur may be larger than the heat of combustion of the sulfur-containing compounds in the MSW used in this study. Another interesting point that should be mentioned is that the coefficients of the sulfur term in the Dulong, Steurer, and Scheurer-Kestner equations were 22.5, 25, and 22.5, respectively, while the coefficient for the same term for the model (based on ultimate analysis) developed in this study was -671.8. It should be mentioned that the sulfur content was very low, and consequently the sulfur term, regardless of its sign, contributes to a limited degree to the energy content.

Figures 5, 6, 7, and 8 show the measured and predicted values of the energy content derived from this research and from the Dulong (equation 3A in Table 1), Steuer (equation 3B in Table 1), and Scheurer-Kestner (equation 3C in Table 1) models, respectively. The model developed here ($R^2 = 0.93$) proved to be far more accurate than the Dulong ($R^2 = 0.58$),

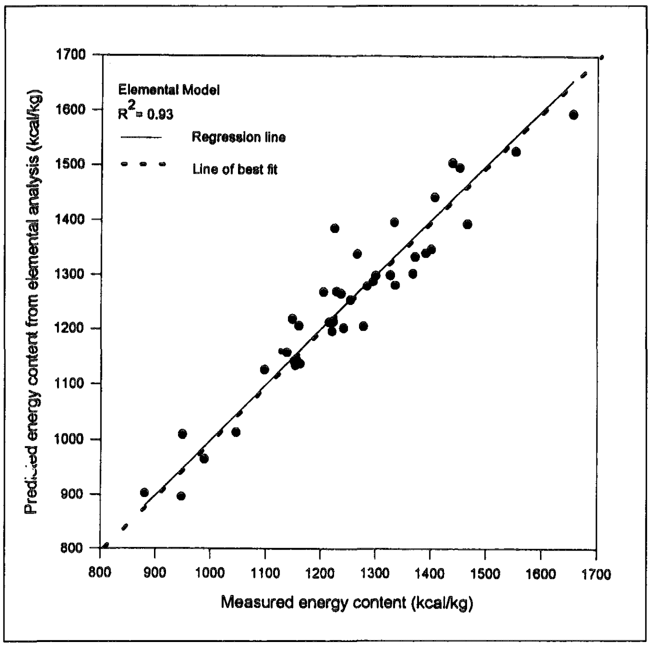

Figure 5. Comparison of measured energy content and the predicted values obtained from elemental model developed by multiple regression analysis.

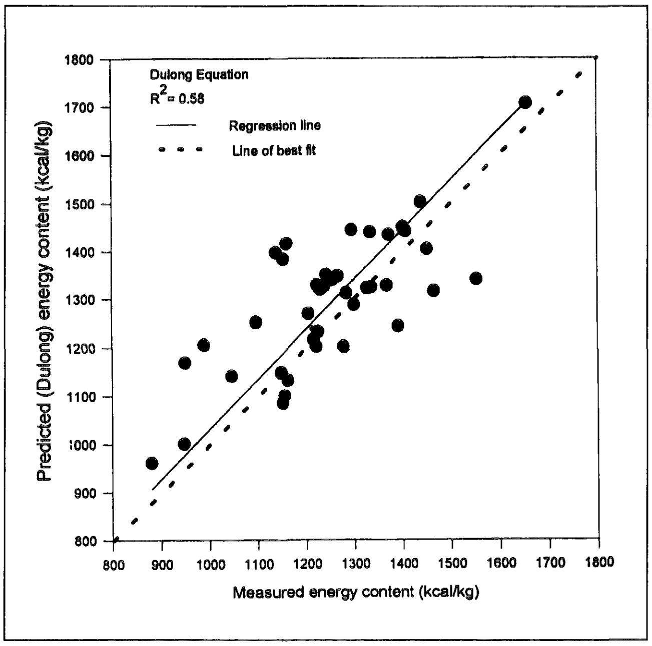

Figure 6. Comparison of measured energy content and the predicted values obtained from Dulong equation (see Table 1).

Scheurer-Kestner ($R^2 = 0.70$), and Steurer ($R^2 = 0.56$) models. The absence of a skew in the data set shown in Figure 5 supports the assumption of a linear relationship between the dependent and independent variables. The Scheurer-Kestner and the Steuer equations tend to overpredict the energy content, while the Dulong model is characterized by considerable scatter.

The multiple regression equations referenced above must be accompanied with the caveat that they are only valid for predicting the energy content of waste with characteristics similar to that of MSW at Kaohsiung City (see Table 2). Their accuracy in predicting the energy content/calorific value of waste with significantly different characteristics is unknown. The authors recommend that individual MSW treatment facilities use regression analysis to develop waste specific models.

Proximate Analysis Models

Attempts to develop a regression model from proximate analysis data were not successful. The stepwise regression

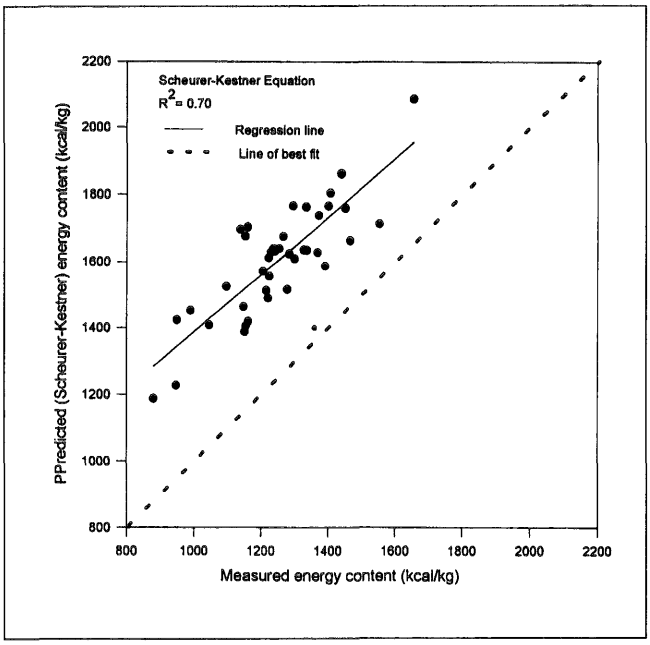

Figure 7. Comparison of measured energy content and predicted values obtained from Scheurer-Kestner equation (see Table 1).

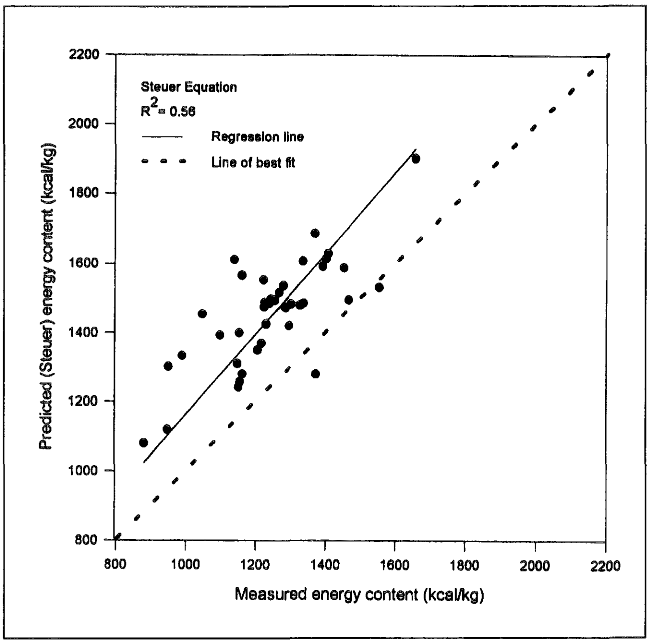

Figure 8. Comparison of measured energy content and predicted values obtained from Steuer equation (see Table 1).

protocol indicated that volatile matter did not contribute significantly to the energy content, and the model did not include this parameter as a predictor variable. This made the model very difficult to defend from a physical standpoint, because volatile matter is the primary source of the energy content.

CONCLUSIONS

- The multiple regression models derived from physical composition analysis and from ultimate analysis accurately predicted the energy content of MSW from Kaohsiung City.
- The regression model based on physical composition was superior to the conventional equation (also based on physical composition) developed by other researchers. The conventional equation had poor accuracy and substantially overpredicted the energy content.
- The model developed in this work based on ultimate analysis was also more accurate than equations/models developed by other researchers.
- Modeling efforts on the proximate analysis database were not successful.

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