Instructions: This homework assignment covers **Chapter 2** of the course notes. On each part, there is opportunity for partial credit, so show all of your work and explain all of your reasoning. No work/no explanation means no credit even if your answer is correct. If you use R to answer any part or to check your work, please include all code and output as attachments. Do not just write out the code you used.

1. A group of 5 applicants for a pair of identical jobs consists of 3 men and 2 women. The employer plans to select 2 of the 5 applicants for the jobs. Let S denote the sample space of all possible pairs chosen. In writing out the sample space, denote the men by M_1, M_2, and M_3. Denote the women by W_1 and W_2. Therefore, the pair (M_1, M_2) means that the first two men are selected for the jobs.

(a) Write out the complete sample space. Does ordering of the selection matter? That is, should we distinguish between (M_1, M_2) and (M_2, M_1)?

Define the following events:

$$\begin{aligned} A &= \{\text{exactly one man is selected}\} \\ B &= \{\text{at least one woman is selected}\}. \end{aligned}$$

(b) List all of the outcomes in A and B and assign probabilities to each event. What assumptions are you making?
(c) Are A and B mutually exclusive? Explain why or why not.
(d) Are A and B independent events? Using numerical calculations, show they are or show they are not.

2. Patients arriving at a hospital outpatient clinic can select one of three stations for medical care. Physicians are assigned randomly to the stations, so assume the patients have no preference which station they are assigned to.

Three patients arrive at the clinic and their selection of the stations is observed. For example, the outcome $(1, 1, 1)$ means that each patient selected station 1. The outcome $(1, 1, 2)$ means the first two patients selected station 1 and the third patient selected station 2, and so on.

(a) Write out the complete sample space. How many outcomes are in the sample space?
(b) Define the event

$$A = \{\text{each station receives one patient}\}.$$

List the outcomes in A and assign the probability $P(A)$. What assumptions are you making?

3. A student prepares for an exam by studying a list of 10 problems. She can solve 7 of them. The instructor puts the exam together by selecting 5 of the 10 problems from the list at random. What is the probability the student will be able to solve all 5 problems on the exam?

4. An experimenter wishes to investigate the effects of three variables−pressure, temperature, and the type of catalyst−on the yield in a refining process. If the experimenter intends to use

- three settings of pressure (100 psi, 125 psi, and 150 psi),

- three settings of temperature (20 deg C, 25 deg C, and 30 deg C), and
- two types of catalysts (C_1 and C_2),

how many experimental runs will be needed to run all possible combinations of pressure, temperature, and type of catalyst? For example, one combination is using catalyst C_1 at a pressure of 100 psi and temperature 20 deg C.

5. An oil exploration company currently has two active projects, one in Asia and one in North America. Let A be the event the one in Asia is successful and let B be the event the one in North America is successful. Seventy percent of past projects in Asia and forty percent of past projects in North America have been successful. Assume A and B are independent.

(a) What is the probability at least one of the two projects will be successful?
(b) What is the probability neither project will be successful?
(c) What is the probability exactly one of the two projects will be successful?
(d) If the Asian project is not successful, what is the probability the North American project is not successful?

6. Articles coming through an inspection line are visually inspected by two successive inspectors. When a defective article comes through the line, the probability it gets by the first inspector is 0.1. The second inspector, on average, will "miss 5 out of 10 of the defective items that get past the first inspector." What is the probability a defective items gets by both inspectors?

7. A large company trains operators who do certain tasks on a production line. Those operators who take the training course are able to meet their production quotas 90 percent of the time. Operators who do not take the training course meet their quotas 60 percent of the time. Seventy percent of the operators have taken the training course.

(a) Define relevant events (there are 2) and interpret the three percentages above in terms of probabilities using appropriate notation. Two of these percentages refer to conditional probabilities.
(b) Find the probability an operator has taken the training course and meets his production quota.
(c) Find the probability an operator meets his production quota.
(d) Find the conditional probability an operator has taken the training course, given that he meets his production quota.
(e) Are the two events you defined in part (a) independent? Explain.

8. A bin contains three components from Supplier A, four from Supplier B, and five from Supplier C. If four of the components are randomly selected for testing, what is the probability each supplier would have at least one component tested?

9. A population of voters contains 40% Republicans and 60% Democrats. It is reported that 30% of the Republicans and 70% of the Democrats favor an election issue. A person is selected

at random from this population and favors the issue in question. Find the (conditional) probability this person is a Democrat.

10. Prove the complement rule for conditional probabilities:

$$P(A'|B) = 1 - P(A|B).$$