1. (a) The sample space is

$$S = \{(M_1, M_2), (M_1, M_3), (M_1, W_1), (M_1, W_2), (M_2, M_3), (M_2, W_1), (M_2, W_2), (M_3, W_1), (M_3, W_2), (W_1, W_2)\}.$$

There are

$$n_S = \binom{5}{2} = 10$$

outcomes in S. We are choosing 2 applicants from 5. The order is not important because the jobs are identical.

(b) Identify the outcomes in S which satisfy each event:

$$\begin{aligned} A &= \{(M_1, W_1), (M_1, W_2), (M_2, W_1), (M_2, W_2), (M_3, W_1), (M_3, W_2)\} \\ B &= \{(M_1, W_1), (M_1, W_2), (M_2, W_1), (M_2, W_2), (M_3, W_1), (M_3, W_2), (W_1, W_2)\}. \end{aligned}$$

Assuming each outcome in S is equally likely, we have

$$P(A) = \frac{n_A}{n_S} = \frac{6}{10} \quad \text{and} \quad P(B) = \frac{n_B}{n_S} = \frac{7}{10}.$$

(c) Note that

$$A \cap B = \{(M_1, W_1), (M_1, W_2), (M_2, W_1), (M_2, W_2), (M_3, W_1), (M_3, W_2)\}$$

is not empty. Therefore, A and B are not mutually exclusive.

(d) A and B are not independent either. Note that

$$P(A \cap B) = \frac{6}{10}.$$

This does not equal

$$P(A)P(B) = \frac{6}{10} \times \frac{7}{10} = \frac{42}{100}.$$

2. (a) There are 3 stations patient 1 can select. There are 3 stations patient 2 can select. There are 3 stations patient 3 can select. From the multiplication rule of counting, there are

$$n_S = 3 \times 3 \times 3 = 27$$

different outcomes in the sample space. These are listed below:

$$\begin{aligned} S = \{&(1,1,1), (1,1,2), (1,1,3), (1,2,1), (1,2,2), (1,2,3), (1,3,1), (1,3,2), (1,3,3), \\ &(2,1,1), (2,1,2), (2,1,3), (2,2,1), (2,2,2), (2,2,3), (2,3,1), (2,3,2), (2,3,3), \\ &(3,1,1), (3,1,2), (3,1,3), (3,2,1), (3,2,2), (3,2,3), (3,3,1), (3,3,2), (3,3,3)\}. \end{aligned}$$

(b) You could go through and list all of the outcomes in A. All we really need is n_A, the number of outcomes in A, and we can count these. For each station to receive one patient, we need the outcomes above which list 1, 2, and 3 in any order; e.g., $(1, 2, 3)$, $(1, 3, 2)$, and so on. There are

$$n_A = 3! = 6$$

such outcomes because there are $3! = 6$ ways to permute the integers 1, 2, and 3. Assuming each outcome in S is equally likely, we have

$$P(A) = \frac{n_A}{n_S} = \frac{6}{27} = \frac{2}{9}.$$

3. There are

$$n_S = \binom{10}{5} = 252$$

different exams the instructor could put together. Define the event

$$A = \{\text{student can solve all 5 problems}\}.$$

How many of these exams will contain problems where the student can solve all of them? To answer this, think of the following. We need to

- select 5 problems from the 7 the student knows: $n_1 = \binom{7}{5} = 21$
- select 0 problems from the 3 the student does not know: $n_2 = \binom{3}{0} = 1$.

By the multiplication rule of counting, we have

$$n_A = n_1 \times n_2 = 21 \times 1 = 21$$

different exams where the student can solve all the problems. Assuming each of the exams is equally likely, the probability is

$$P(A) = \frac{n_A}{n_S} = \frac{21}{252} = \frac{1}{12}.$$

4. Think of choosing one "level" from each of the three factors in sequence:

- there are $n_1 = 3$ different settings (levels) of pressure
- there are $n_2 = 3$ different settings (levels) of temperature
- there are $n_3 = 2$ different catalyst types (levels).

By the multiplication rule of counting, there are

$$3 \times 3 \times 2 = 18$$

different combinations of the levels of pressure, temperature, and catalyst type. The terms "factor" and "level" are used in the design of experiments.

5. Define the events

$$\begin{aligned} A &= \{\text{project in Asia is successful}\} \\ B &= \{\text{project in North America is successful}\}. \end{aligned}$$

We are given $P(A) = 0.7$ and $P(B) = 0.4$. We also assume A and B are independent.

(a) We want

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= P(A) + P(B) - P(A)P(B) \qquad \text{(independence)} \\ &= 0.7 + 0.4 - (0.7)(0.4) = 0.82. \end{aligned}$$

(b) "Neither project is successful" means $A' \cap B'$ has occurred. Recall $A' \cap B' = (A \cup B)'$ by DeMorgan's Law. Therefore,

$$P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.82 = 0.18.$$

(c) "Exactly one is successful" means that either $A \cap B'$ or $A' \cap B$ occurred. These are mutually exclusive events so we want

$$\begin{aligned} P(A \cap B') + P(A' \cap B) &= P(A)P(B') + P(A')P(B) \qquad \text{(independence)} \\ &= (0.7)(0.6) + (0.3)(0.4) = 0.54. \end{aligned}$$

(d) Technically, we want $P(B'|A')$. However, A and B are independent events. Therefore, regardless of whether A occurs or not, it doesn't affect how we assign probability to B. We have

$$P(B'|A') = P(B') = 1 - P(B) = 1 - 0.4 = 0.6.$$

6. Define the events

$$\begin{aligned} A_1 &= \{\text{defective item gets by inspector 1}\} \\ A_2 &= \{\text{defective item gets by inspector 2}\}. \end{aligned}$$

We are given $P(A_1) = 0.1$ and $P(A_2|A_1) = 0.5$. We want

$$P(A_1 \cap A_2) = P(A_2|A_1)P(A_1) = (0.5)(0.1) = 0.05.$$

This is the multiplication rule for probabilities.

7. (a) There are two relevant events here:

$$\begin{aligned} A &= \{\text{operator takes training course}\} \\ B &= \{\text{operator meets quota}\}. \end{aligned}$$

We are given $P(B|A) = 0.9$, $P(B|A') = 0.6$, and $P(A) = 0.7$.
(b) We want

$$P(A \cap B) = P(B|A)P(A) = (0.9)(0.7) = 0.63.$$

(c) Use LOTP. We want

$$\begin{aligned} P(B) &= P(B|A)P(A) + P(B|A')P(A') \\ &= (0.9)(0.7) + (0.6)(0.3) = 0.81. \end{aligned}$$

(d) We want

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.63}{0.81} \approx 0.51.$$

(e) Notice how $P(B) = 0.81$ and $P(B|A) = 0.9$. That is, knowledge that A occurs changes how we assign probability to B. Therefore, A and B cannot be independent. You could also show

$$P(A \cap B) \neq P(A)P(B).$$

8. There are 12 distinct components total, and we are selecting 4 of them for testing. There are

$$n_S = \binom{12}{4} = 495$$

ways this can be done. Outcomes in the sample space can be conceptualized as

$$(\_\_ \ \_\_ \ \_\_ \ \_\_).$$

One possible outcome is

$$(\underline{A_1} \ \underline{A_2} \ \underline{B_4} \ \underline{C_3}).$$

This outcome identifies 2 components selected from Supplier A, one from Supplier B, and one from Supplier C. We are treating the components as distinct objects (hence, the use of subscripts to differentiate components within supplier). However, the ordering is not important. That is, we are not distinguishing the outcome above from

$$(\underline{A_2} \ \underline{A_1} \ \underline{C_3} \ \underline{B_4}), \quad \text{say.}$$

Define the event

$$A = \{\text{each supplier will have at least one component tested}\}.$$

We need to count the number of outcomes like the one above−where components from Supplier A, B, and C are all represented. This means one supplier will have 2 components in the sample (of 4) that is selected.

- If Supplier A is represented twice in the sample, there are

$$\binom{3}{2}\binom{4}{1}\binom{5}{1} = 60 \text{ outcomes.}$$

- If Supplier B is represented twice in the sample, there are

$$\binom{3}{1}\binom{4}{2}\binom{5}{1} = 90 \text{ outcomes.}$$

- If Supplier C is represented twice in the sample, there are

$$\binom{3}{1}\binom{4}{1}\binom{5}{2} = 120 \text{ outcomes.}$$

Therefore, there are

$$n_A = 60 + 90 + 120 = 270$$

different outcomes in A. Assuming each outcome is equally likely, we have

$$P(A) = \frac{n_A}{n_S} = \frac{270}{495} \approx 0.55.$$

9. Define the following two events:

$$\begin{aligned} A &= \{\text{person selected is Democrat}\} \\ B &= \{\text{person selected agrees with issue}\}. \end{aligned}$$

We are given $P(A) = 0.6$, $P(B|A) = 0.7$ and $P(B|A') = 0.3$. We want $P(A|B)$. Use Bayes' Rule. We have

$$\begin{aligned} P(A|B) &= \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')} \\ &= \frac{(0.7)(0.6)}{(0.7)(0.6) + (0.3)(0.4)} \approx 0.78. \end{aligned}$$

10. Use the definition of conditional probability. Recall

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ P(A'|B) &= \frac{P(A' \cap B)}{P(B)}. \end{aligned}$$

Therefore,

$$P(A|B) + P(A'|B) = \frac{P(A \cap B)}{P(B)} + \frac{P(A' \cap B)}{P(B)} = \frac{P(A \cap B) + P(A' \cap B)}{P(B)}.$$

Now, note that $A \cap B$ and $A' \cap B$ are mutually exclusive events (draw a Venn diagram) and

$$P(A \cap B) + P(A' \cap B) = P(B).$$

This shows

$$P(A|B) + P(A'|B) = \frac{P(B)}{P(B)} = 1$$

and hence the result follows.