Instructions: This homework assignment covers **Chapter 3** of the course notes. On each part, there is opportunity for partial credit, so show all of your work and explain all of your reasoning. No work/no explanation means no credit even if your answer is correct. If you use R to answer any part or to check your work, please include all code and output as attachments. Do not just write out the code you used.

1. A textile engineer uses the probability mass function (pmf)

x	0	1	2	3	4
$p_X(x)$	0.40	0.38	0.16	0.05	0.01

as a model for

$$X = \text{the number of imperfections in a 10-meter piece of synthetic fabric.}$$

(a) Calculate $E(X)$. Plot the pmf of X and locate where $E(X)$ is on the horizontal axis.
(b) Find the cumulative distribution function (cdf) of X.

2. Results from the 2011 National Youth Risk Behavior Survey were used to formulate the probability mass function (pmf) for

$$X = \text{the number of days per week a student is physically active.}$$

Being "physically active" was defined as "engaging in physical activity for at least 60 minutes." This pmf, which was formulated for the population of all high school students, is below:

x	0	1	2	3	4	5	6	7
$p_X(x)$	0.15	0.08	0.10	0.11	0.10	0.12	0.07	0.27

(a) What percentage of students in this population are physically active for at least two days per week?
(b) Calculate the standard deviation of X.

3. On a recent trip to the SC DMV, I asked a manager to estimate what percentage of SC drivers arrive to renew their driver's license with one that is currently expired. She informed me that 20 percent of all license renewals were of this type.

(a) On a given day, a SC DMV office sees 150 customers wanting to renew their license. Let X denote the number of customers (out of 150) whose license is currently expired. What is the distribution of X?
(b) Let Y denote the number of customers observed to find the first one with an expired license. What is the distribution of Y?
(c) Let W denote the number of customers observed to find the third one with an expired license. What is the distribution of W?
(d) In parts (a)-(c), you are assuming the Bernoulli trial assumptions hold. State what these are in this example. *Hint:* Think of each customer seeking renewal as a "trial."

4. An electronic product contains 40 integrated circuits. The probability that any circuit is defective is 0.01, and the circuits are independent. The product operates only if there are no defective circuits.

(a) What is the probability that the product operates? Answer this question using the binomial distribution, letting X denote the number of defective circuits (out of 40). Are the Bernoulli trial assumptions reasonable here?
(b) Calculate $E(X)$ and $V(X)$ assuming the binomial model is reasonable.

5. An array of 30 LED bulbs is used in an automotive light. The probability any one bulb is defective is 0.001 and defective bulbs occur independently.

(a) Find the probability an automotive light has two or more defective bulbs.
(b) Engineers plan to observe automotive lights until they find the first one with two or more defective bulbs. Find the mean number of lights they will observe.

6. Monitoring populations of mosquito vectors is an important part of agricultural and public health risk assessment. Some vector-borne pathogens, such as West Nile Virus (WNV), are potentially so serious that observing just one infected mosquito may be enough for public health officials to declare a state of emergency.

Researchers estimate the proportion of WNV-infected mosquitos in Jefferson County, Florida, is $p = 0.0005$.

(a) In this county, let X denote the number of mosquitos researchers must test to find the first one infected with WNV. Find $P(X \geq 100)$. *Hint:* You can do this part "by hand" if you remember finite geometric sums; otherwise, you can use R.
(b) Researchers in this county will continually test mosquitos as in part (a). A state of emergency will be declared when the researchers find the fifth mosquito infected with WNV. What is the mean number of mosquitos that must be tested to declare this emergency?

7. A police officer at SLED who specializes in illegal narcotics has 40 identical looking packages of white powder. Ten of these packages contain cocaine and 30 do not. Four packages are randomly selected (without replacement) and are tested.

(a) What is the probability exactly 2 of the four packages contain cocaine?
(b) What is the probability at most 2 of the four packages contain cocaine?

8. An automobile manufacturer is concerned about a fault in a braking mechanism of a particular model. The fault can, on rare occasions, cause a catastrophe at high speed. Suppose the distribution of X, the number of cars per year that will experience the catastrophe, is Poisson with mean $\lambda = 2.5$.

(a) In a given year, find the probability

- no cars will experience the catastrophe

- 3 or more cars will experience the catastrophe.

(b) The cost to the manufacturer associated with a catastrophe is potentially enormous. Risk management experts have estimated the yearly cost associated with X catastrophes (in \$1000s) is

$$C = 150 + 1000X + 0.1X^2.$$

Find $E(C)$, the expected cost per year. *Hint:* Remember that $E(X^2) \neq [E(X)]^2$.