Instructions: This homework assignment covers **Chapter 4** of the course notes. On each part, there is opportunity for partial credit, so show all of your work and explain all of your reasoning. No work/no explanation means no credit even if your answer is correct. If you use R to answer any part or to check your work, please include all code and output as attachments. Do not just write out the code you used.

1. A critical factor in solid missile fuel is the particle size distribution. Significant problems occur if the particle sizes are too large. From production data in the past, it has been determined the probability density function (pdf) of the particle size (X, in micrometers) is

$$f_X(x) = \begin{cases} \dfrac{3}{x^4}, & x > 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Prepare a graph of $f_X(x)$.
(b) Find the probability a particle exceeds 2.5 micrometers in size.
(c) The cumulative distribution function (cdf) of X is

$$F_X(x) = \int_{-\infty}^{x} f_X(t)dt = \begin{cases} 0, & x \leq 1 \\ 1 - \dfrac{1}{x^3}, & x > 1 \end{cases}.$$

What proportion of particles will be between 1.5 and 3 micrometers in size?

2. The pH of water samples from lakes in a specific region is a continuous random variable X with probability density function (pdf)

$$f_X(x) = \begin{cases} \dfrac{3}{8}(7-x)^2, & 5 \leq x \leq 7 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Prepare a graph of $f_X(x)$.
(b) Find the mean and variance of X.

3. The length of time until failure (in years) for a transistor is a continuous random variable X with probability density function (pdf)

$$f_X(x) = \begin{cases} 2xe^{-x^2}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Prepare a graph of $f_X(x)$.
(b) The cumulative distribution function (cdf) of X is

$$F_X(x) = \int_{-\infty}^{x} f_X(t)dt = \begin{cases} 0, & x < 0 \\ 1 - e^{-x^2}, & x \geq 0. \end{cases}$$

Find $\phi_{0.90}$, the 90th percentile of this distribution and interpret what this means.

4. Chemical pollutants are substances that contaminate the environment, posing risks to human health and ecosystems. Suppose the concentration of a particular pollutant, X (measured in parts per million, ppm), has an exponential distribution with $\lambda = 0.03$.

(a) What is the probability the concentration is greater than 50 ppm?
(b) Find the mean and median concentration. Why is the mean larger?

5. The time to death (X, measured in days) for a group of patients with stage 4 colon cancer follows a gamma distribution with $\alpha = 2.5$ and $\lambda = 0.0025$.

(a) Find the proportion of patients who will die in less than one year. Regard one year as 365 days.
(b) Ten patients are selected at random from this group. What is the probability at least 2 of them will die in less than one year? *Hint:* Think of each patient as a "trial," and use the binomial distribution.

6. Small aircraft arrive at a local airport according to a Poisson process with a mean $\lambda = 4.5$ landings per hour.

(a) What is the distribution of the time until the first aircraft lands? the 3rd aircraft?
(b) What is the probability there are no aircraft landing in any given hour? Answer this question in two ways; one using the Poisson distribution directly and one using the exponential distribution.

7. PM10 refers to particulate matter (PM) with a diameter of 10 micrometers or less, making it "inhalable" and small enough to enter the lungs. These tiny particles can include smoke, soot, dust, metals, and acids, and can originate from various human-made sources like industrial processes, vehicle exhaust, and domestic heating, as well as natural sources such as dust storms. Excessive exposure to PM10 can lead to adverse health effects.

A 2012 research article in *Atmospheric Chemistry and Physics* examined air quality data from sensors in Pheonix, AZ. Suppose the authors model

$$X = \text{daily PM10 level for a centrally located sensor (in micrometers per m}^3\text{)}$$

using a normal distribution with mean $\mu = 52.5$ and standard deviation $\sigma = 12.5$.

(a) What is the probability the daily PM10 level exceeds 75 micrometers per m^3?
(b) What proportion of days will the PM10 level be between 40 and 65 micrometers per m^3?
(c) Find the 99th percentile of this distribution and interpret what this means.