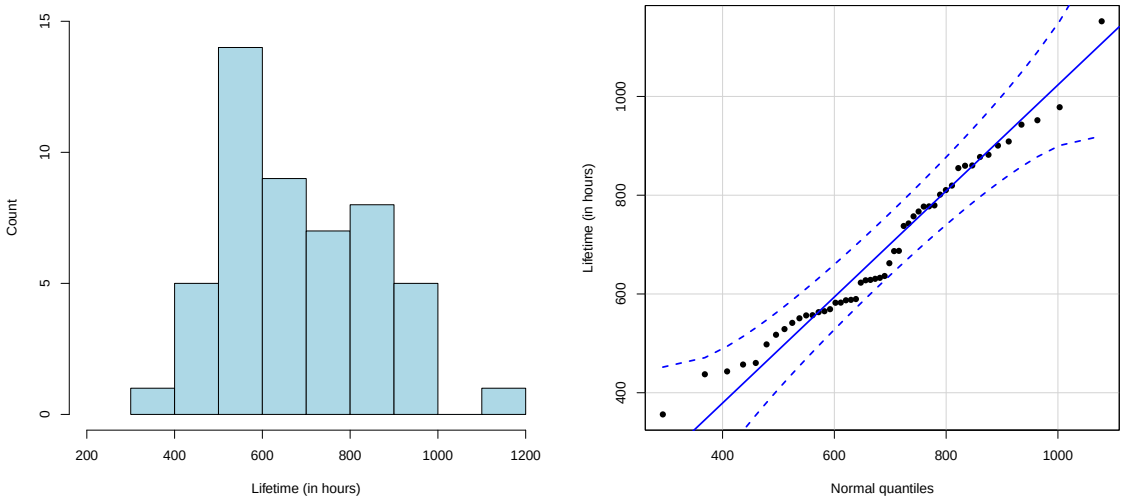
Instructions: This homework assignment covers **Chapter 7** of the course notes. On each part, there is opportunity for partial credit, so show all of your work and explain all of your reasoning. No work/no explanation means no credit even if your answer is correct. If you use R to answer any part or to check your work, please include all code and output as attachments. Do not just write out the code you used.

1. A new brand of light bulb is advertised as "having an average lifetime of 750 hours." The price of the bulbs is favorable, so a customer has decided to go ahead with a purchase arrangement unless it can be demonstrated the population mean lifetime μ is smaller than the "750 hours" advertised figure. A random sample of $n = 50$ bulbs was selected and the lifetime (X, measured in hours) of each bulb was measured. The following R output gives the sample mean and sample standard deviation of the 50 lifetimes:

```
> mean(bulbs)
[1] 685.7
> sd(bulbs)
[1] 168.8
```

A histogram and normal quantile-quantile (qq) plot are shown below.



(a) Calculate a 95% confidence interval for the population mean lifetime μ. Interpret the interval. How would you advise the customer to proceed?
(b) What are the assumptions associated with your confidence interval in part (a)? Discuss whether you think they are reasonable.
(c) In addition to the mean lifetime, the customer also wants an interval estimate of the population standard deviation. Calculate a 95% confidence interval for σ and interpret it. Do you have any concerns about the underlying assumptions being met for this interval?

2. The United States Army recently commissioned a study to assess how deeply a bullet penetrates ceramic body armor. In the study, a cylindrical clay model was layered under an armor vest. A projectile was then fired, causing an indentation in the clay. The deepest impression in the clay was measured as an indication of survivability of someone wearing the

armor. The data below are measurements of X, the deepest indentation (in mm) for a sample of $n = 83$ clay models. Measurements were made using a manually controlled digital caliper.

22.4	23.6	24.0	24.9	25.5	25.6	25.8	26.1	26.4	26.7	27.4	27.6	28.3
29.0	29.1	29.6	29.7	29.8	29.9	30.0	30.4	30.5	30.7	30.7	31.0	31.0
31.4	31.6	31.7	31.9	31.9	32.0	32.1	32.4	32.5	32.5	32.6	32.9	33.1
33.3	33.5	33.5	33.5	33.5	33.6	33.6	33.8	33.9	34.1	34.2	34.6	34.6
35.0	35.2	35.2	35.4	35.4	35.4	35.5	35.7	35.8	36.0	36.0	36.0	36.1
36.1	36.2	36.4	36.6	37.0	37.4	37.5	37.5	38.0	38.7	38.8	39.8	41.0
42.0	42.1	44.6	48.3	55.0								

(a) Calculate a 95% confidence interval for the population mean deepest indentation μ. Interpret the interval. Check the normality assumption using a qq plot and discuss.
(b) A co-worker of yours looks at your interval in part (a) and says,

> *"I'm confused. Ninety-five percent of the measurements above should be in your interval, but far fewer are."*

What is your co-worker getting wrong?
(c) Calculate a 95% confidence interval for the population standard deviation σ. Interpret the interval.
(d) What is the length of your confidence interval in part (a)? Suppose the Army wants to perform a new study and needs to determine how many clay models to sample. The new study will use

- a 99% confidence interval for the population mean μ
- an interval length equal to half the length of your confidence interval in part (a).

How many clay models would the new study need?

3. PRAMS, the Pregnancy Risk Assessment Monitoring System, is a surveillance project of the Centers for Disease Control and Prevention and state health departments. In a recent PRAMS survey, 250 women who had given birth were asked about their smoking habits. Smoking during the last 3 months of pregnancy was reported by 31 of those sampled.
(a) What is the population here? Give a reasonable answer.
(b) Find a 95% confidence interval for the population proportion p of women who smoked during the last 3 months of pregnancy. Interpret your interval and discuss your findings. Check the rules of thumb needed to ensure your confidence interval is valid.
(c) Epidemiologists want to use the results from the PRAMS survey to design a larger study, where they will estimate p with a 99% confidence interval. They would like the margin of error associated with the confidence interval to be no more than 0.02 (i.e., a total interval length of no more than 0.04). Determine the sample size needed to meet these requirements.